Senior Colloquium: *History of Mathematics*

Math 400 Spring 2020 Fowler 310 T 1:30pm - 2:55pm

http://sites.oxy.edu/ron/math/400/20/

Worksheet 9: **Tuesday April 14**

TITLE Twenty-First Century Mathematicians
CURRENT READING Katz, 874-903 (§25.1-25.3); Boyer 548-586 (§23)
NEXT READING None!

Artur Avila (Cordeiro de Melo) (b. 1979) is the first Latin American to win the Fields medal (in 2014). Avila was born in Rio de Janeiro, Brazil and at age 16 won a gold medal at the 1995 International Mathematical Olympiad. He earned his Ph.D. at age 22 from *Instituto Nacional de Matemática Pura e Aplicada* (IMPA) in the area of dynamical systems. In 2018 he was appointed full professor at the University of Zurich. He had made significant contributions to chaos theory and dynamical systems.

Manjul Bhargava (b. 1974) is a 2014 Fields medalist and Canadian-American mathematician with academic appointments at multiple prestigious research institutions (Princeton University, Leiden University, Indian Institute of Technology Bombay, etc). He received his PhD from Princeton in 2001 (under advisor Andrew Wiles). His Ph.D. thesis extended some results of Gauss on binary quadratic forms. His undergraduate thesis from Harvard in 1996 introduced a generalization of the factorial function and was awarded the Morgan Prize (the highest honor to an undergraduate student in mathematics in the United States). He has made numerous, extremely significant contributions to number theory and became the third youngest ever full professor at Princeton in 2003. His Erdos number is 3.

Ingrid Daubechies (b. 1954) is a Belgian-born theoretical physicist who is most well-known for her contribution to and popularization of "wavelets." She earned her Ph.D. in theoretical physics in 1980 in Brussels and at AT&T Bell Labs she constructed a continuous wavelet which had special properties that made it extremely useful in numerous applications, such as digital signal process. She has become one of the most highly cited mathematicians in the world and has received numerous prizes, such as the MacArthur in 1992. She was the first female full Professor in Princeton's mathematics department and the third woman in history to give the prestigious Gibbs lecture at the annual Joint Mathematics Meetings.

Benoit Mandelbrot (1924-2010)
Mandelbrot is most well-known for his coining of the term "fractal" in 1975 and the discovery/creation of the Mandelbrot set; these are examples of his "theory of roughness" which he used to mathematically describe real-world phenomena such as the coastline of Britain, the shapes of mountains and the path of lightning in his book *The Fractal Geometry of Nature* (1982). He was born in a Jewish family in Poland who moved to France before World War II. After Mandelbrot received his Mathematical Sciences PhD from the University of Paris in 1952, he made his way to the U.S. where he spent a year at the Institute for Advanced Study in Princeton sponsored by von Neumann and eventually 35 years working at IBM. In addition to his work in mathematics, Mandelbrot published papers in fluid dynamics, information theory, physics and economics.

Maryam Mirzakhani (1977-2017)
Mirzakhani is the first woman to the Field Medal, the most prestigious prize in mathematics. The Fields Medal is only available to research mathematicians who are 40 or younger and is awarded every four years at the International Congress of Mathematicians organized by the International Mathematics Union. Mizakhani received the Fields Medal for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces" at the 2014 ICM in Seoul, South Korea. Mirzakhani was already diagnosed with breast cancer at the time she won the Fields Medal and died 3 years later at age 40. Mirzakhani has become a role model and inspiration for women in mathematics, with her birthday May 12 "International Women in Mathematics Day."

John Simons (b. 1938) is an American mathematician, investor and billionaire philanthropist who is one of the richest men in the United States (with total wealth estimated over $20 billion in 2019). After graduating from the Massachusetts Institute of Technology in 1958 with a mathematics bachelor's degree, Simons received his Ph.D. in mathematics from University of California Berkeley three years later at the age of 23. He was the chair of the mathematics department at Stony Brook University from 1968 to 1978. He founded a private hedge fund named Renaissance Technologies in 1982 that was so successful he has often been called "the greatest investor on Wall Street." In 1994, he formed the Simons Foundation, which has become one of the most prominent private supporters of mathematical research and education in the United States. One of the first grants from the Simons Foundation was $25 million to create Math for America, an organization that provides mentoring, training and financial support to American students who graduate with STEM degrees to become high school and middle school teachers.

Stephen Smale (b. 1930)
Smale is an American mathematician (and 1966 Fields medalist) who is most well-known for his contributions to topology, dynamical systems, chaos theory and mathematical economics. Smale was a longtime professor at University of California Berkeley. His academic career in mathematics began inauspiciously, with mediocre grades at the undergraduate and graduate level, although he earned his PhD from the University of Michigan in 1957. Soon afterwards he stunned the mathematical community with a proof of the Poincaré conjecture for all dimensions 5 or greater in 1961 and also introduced the Smale horseshoe as a canonical example of chaos in 1967. Smale produced a list of 18 problems in 1998 that mathematicians should attempt to solve in the twenty-first century (similar to Poincaré's 23 problems for the twentieth century). Several of Smale's problems became Millennium Prize Problems which are the 7 problems the Clay Mathematics Institute has stated it will provide $1 million to anyone who solves them.

Terence Tao (b. 1975) is an Australian-American Fields medalist who became the youngest ever full professor at University of California Los Angeles in 1999 at age 24. Tao was a famous child prodigy, competing in the International Mathematics Olympiad at age 10, and is the youngest person ever to win bronze (1986), silver (1987) and gold (1988) medals. At age 16 he received his bachelor's and master's degrees simultaneously from Flinders University in Adelaide; he earned his PhD from Princeton at age 21 and 10 years later was awarded the Fields Medal and MacArthur "genius grant" at age 31. He has published well over 300 research papers and 17 books, in a wide array of mathematical fields, with various co-authors. His Erdos number is 2.

Karen Uhlenbeck (b. 1942)
Uhlenbeck is the first woman to win the Abel Prize, one of the most prestigious prizes in mathematics; she did so in 2019 for "her pioneering achievements in geometric partial differential equations, gauge theory, and integrable systems, and for the fundamental impact of her work on analysis, geometry and mathematical physics" (Abel Prize commendation, https://www.abelprize.no/c73996/binfil/download.php?tid=74095). Uhlenbeck is most well-known for her contributions to the field of geometric analysis, which involves using tools from differential geometry to provide insights to differential equations. She is also known for her commitment to broadening the participation by women in mathematics and is the co-founder of the Park City Mathematics Institute and the Women and Mathematics Program at the Institute for Advanced Study.

Andrew Wiles (b. 1953)
Wiles gained worldwide fame and a permanent place in the history of mathematics by completing a proof of Fermat's Last Theorem (which is that there does not exist any distinct positive integers a,b,c that satisfy the equation $a^n+b^n=c^n$ for $n>2$). Fermat had written in 1637 that he had a proof of this statement that "this margin is too narrow to contain." However, it took 358 years until a proof was published that was acceptable to the mathematics community (in 1995 by Andrew Wiles). Wiles is a British-born mathematician who is a longtime Princeton professor and through his proof of Fermat's Last Theorem has contributed multiple, major advances to number theory. He has been awarded every major prize in academia, such as the MacArthur "genius" award (in 1997), the 2016 Abel Prize and the Clay Research award (in 1999). Notably Wiles' accolades do not include the Fields medal, because he did not finish his work on his proof until after he turned 40.

The Millennium Problems
There are seven problems announced by the Clay Mathematics Institute on May 24, 2000 that would earn $1 million for the individual or team who publish a solution. Only one has been solved so far.

P versus NP
This is one of the most prominent and important unsolved problems in computer science. It asks whether every problem whose solution can be *verified* easily can also be *solved* easily. The formal statement of the *P vs.* NP problem was published in 1971 by American computer scientist **Paul Cook**; it builds on work by John von Neumann, Kurt Godel and John Nash. In general, one can place computable problems in two different classes of problems (i.e. complexity classes): the class of problems where the computation time required to solve it is proportional to a polynomial function of the input is called "class P" while the class of problems where the answer to the problem can be verified in polynomial time is called "class NP." If *P=NP* this would mean that it is likely that there are "easy" (i.e. polynomial time) solutions to "hard" computational problems (e.g. the traveling salesman problem, the graph coloring problem, the knapsack problem, etc). A *NP-complete* problem is an *NP* problem that can be converted into any other *NP* problem in polynomial time. Surveys of computer scientists show that the vast majority (i.e. 99%) believe that *P≠NP* but Don Knuth is not one of them, he has said that he believes *P=NP* but that the proof of this fact will not lead to significant advances.

The Poincaré conjecture
The Poincaré conjecture ("Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere") was one of Hilbert's 23 Problems announced in 1902. Smale proved the conjecture was true for all dimensions greater than or equal to five in 1961 and **Michael Freedman** (son of Occidental mathematics professor Benedict Freedman!) proved that it was true in four dimensions in 1982. The original (and hardest) version of the conjecture in 3-dimensions was proven to be true by Russian mathematician **Grigori Perelman** (b. 1966) a century after Poincaré's challenge, when Perelman posted a sequence of papers to arXiv.org in 2002-2003. For this, Perelman was awarded the Fields medal in 2006 (which he refused!) and in 2010 the Clay Math Institute declared that he would receive the Clay Millenium Prize of $1 million and Perelman also refused that as well. The notoriously reclusive mathematician is said to have completely abandoned mathematics and is believe to currently live with his mother in St. Petersburg.

Navier–Stokes existence and smoothness problem
The Navier-Stokes equations are a system of nonlinear partial differential equations that model fluid flow.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}(\boldsymbol{x}, t)$$
$$\nabla \cdot \mathbf{v} = 0$$

These equations represent 4 unknown functions (3 components of velocity and the pressure field) in 4 equations (given the conservation of mass represented by the continuity equation for incompressible flow represented by the second expression.) The question is whether there always exist solutions to the Navier-Stokes equations that are smooth and bounded. The official statement of the problem is "**Prove or disprove: In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the Navier–Stokes equations.**" The Navier-Stokes equations are incredibly important in the design and testing of new aircraft, ships and cars.

The Riemann hypothesis
The Riemann hypothesis can be stated very simply as "**The real part of every non-trivial zero of the Riemann zeta function is ½.**" The conjecture was first made by **Bernhard Riemann** (1859) and involves connections between two apparently disparate sections of mathematics: complex analysis and number theory. If the conjecture is true, then it will provide important information about the distribution of the prime numbers, which is a key goal of number theory.
The Riemann zeta function *ζ(s)* is a function of the complex variable *s* can be shown to converge when Re(*s*)<1 and is:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \ldots = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

The three remaining Millennium problems are in such specialized fields of mathematics they are hard to even state in ways that are comprehensible to non-experts.

Yang–Mills existence and mass gap
This Millennium problem is named after Chen Ning Yang and Robert Mills, two mathematical physicists who jointly developed a theory of subatomic particles in the 1950s called Yang-Mills theory. It combines quantum physics and Lie algebra.

The official statement of the problem is:

> **Prove that for any compact simple gauge group *G*, a non-trivial quantum Yang–Mills theory exists on R^4 and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964), Osterwalder & Schrader (1973) and Osterwalder & Schrader (1975).**

The Birch and Swinnerton-Dyer conjecture
This Millennium problem is an open problem in number theory named after two British mathematicians, **Brian Birch** (b. 1931) and **Peter Swinnerton-Dyer** (1927-2018). The official statement of the conjecture is posed by Wiles:

> **The rank of the abelian group *E(K)* of points of *E* is the order of the zero of *L(E, s)* at *s* = 1, and the first non-zero coefficient in the Taylor expansion of *L(E, s)* at *s* = 1 is given by more refined arithmetic data attached to *E* over *K*.**

The Hodge conjecture
This Millennium problem is an open problem in algebraic geometry named after British mathematician W.V.D. Hodge (1903-1975). The official statement of the problem is:

> **Let *X* be a non-singular complex projective manifold. Then every Hodge class on *X* is a linear combination with rational coefficients of the cohomology classes of complex subvarieties of *X*.**

DISCUSSION

What are your thoughts about the nature of the complexity of the mathematics we have to consider as our investigation of the history of mathematics approaches the present?