Senior Colloquium: *History of Mathematics*

Math 400 Spring 2020 Fowler 310 T 1:30pm - 2:55pm

 http://sites.oxy.edu/ron/math/400/20/

Worksheet 1: **Tuesday January 21**

TITLE Introduction to History of Mathematics: From Euler to Uhlenbeck
CURRENT READING Katz, 594; Boyer 406-422; Stillwell 200-202

Homework Assignments due Tuesday January 28 1:30PM
HW #0: Automathography. E-mail me a short (between 250-500 words) "automathography" of yourself. Automathography is a neologism composed of the word autobiography and math, and it is basically a a biography of your mathematical self, written by you. Tell me the answers to (some of) the following questions: What 300-level mathematics courses have you taken? What is your favorite math class so far (and why?) what you find hardest. What are your mathematical interests? What do you plan to do after graduation? Why did you sign up for this course and what do you expect to get out of it? If your aims for taking this course are different from the stated goals in the syllabus, please let me know. If you have any anxieties about this course, or any special problems or needs, let me know. You are encouraged to be creative in your response; don't be pedantic and only answer the questions asked above; they are intended to serve as a guide for your written introduction to me.

SUMMARY
In today's class we shall go through an overview of the class, discuss the syllabus, and begin our discussion of the history of mathematics.

This class is both a mathematics class and a history class. To me that makes this class an interesting challenge to teach, because although I have taught many mathematics classes I have not taught very many history classes.

Epistemology
In history (and other classes) we should always think about the epistemological question "How do we know what we know?"

Historically, this often means that we need to consider the source of the material we are reading and hopefully delve into the provenance of the source.

Mathematically, this often means "showing your work" or depending on lemmas and theorems in order to make a convincing case to the reader that the mathematical statements one is making are true.

From Katz (p. 594):

BIOGRAPHY

Leonhard Euler (1707–1783)

Born in Basel, Switzerland, Euler showed his brilliance early, graduating with honors from the University of Basel when he was fifteen. Although his father preferred that he prepare for the ministry, Euler managed to convince Johann Bernoulli to tutor him privately in mathematics. The latter soon recognized his student's genius and persuaded Euler's father to allow him to concentrate on mathematics. In 1726 Euler was turned down for a position at the University, partly because of his youth. A few years earlier, however, Peter the Great of Russia, on the urging of Leibniz, had decided to create the St. Petersburg Academy of Sciences as part of his efforts to modernize the Russian state. Among the earliest members of the Academy, appointed in 1725, were Nicolaus II (1695–1726) and Daniel Bernoulli (1700–1782), two of Johann's sons with whom Euler had developed a friendship. Although there was no position in mathematics available in St. Petersburg in 1726, they nevertheless recommended him for the vacancy in medicine and physiology, a position Euler immediately accepted. (He had studied these fields during his time at Basel.)

In 1733, due to Nicolaus's death and Daniel's return to Switzerland, Euler was appointed the Academy's chief mathematician. Late in the same year he married Catherine Gsell with whom he subsequently had 13 children. The life of a foreign scientist was not always carefree in Russia at the time. Nevertheless, Euler was able generally to steer clear of controversies, until the problems surrounding the succession to the Russian throne in 1741 convinced him to accept the invitation of Frederick II of Prussia to join the Berlin Academy of Sciences, founded by Frederick I also on the advice of Leibniz. He soon became director of the Academy's mathematics section and, with the publication of his texts in analysis as well as numerous mathematical articles, became recognized as the premier mathematician of Europe. In 1755 the Paris Academy of Sciences named him a foreign member, partly in recognition of his winning their biennial prize competition 12 times.

Ultimately, however, Frederick tired of Euler's lack of philosophical sophistication. When the two could not agree on financial arrangements or on academic freedom, Euler returned to Russia in 1766 at the invitation of Empress Catherine the Great, whose succession to the throne marked Russia's return to the westernizing policies of Peter the Great. With the financial security of his family now assured, Euler continued his mathematical activities even though he became almost totally blind in 1771. His prodigious memory enabled him to perform detailed calculations in his head. Thus, he was able to dictate his articles and letters to his sons and others virtually until the day of his sudden death in 1783 while playing with one of his grandchildren (Fig. 17.7).

Boyer's *A History of Mathematics* says about Euler's 1748 work *Introductio in analysin infinitorum (Introduction to Analysis of the Infinite)*:

> **It may be fairly said that Euler did for the infinite analysis of Newton and Leibniz what Euclid had done for the geometry of Eudoxus and Theaetetus, or what Viete had done for the algebra of al-Khowarizmi and Cardan. Euler took the differential calculus and the method of fluxions and made them part of a more general branch of mathematics which has been known as "analysis"--the study of infinite processes. If the ancient *Elements* was the cornerstone of geometry and the medieval *Al jabr wa'l muqbalah* was the foundation stone of algebra, then Euler's *Introductio in analysin infinitorum* can be thought of as the keystone of analysis.**

Pierre-Simon Laplace (1749-1827) called Euler "the master of us all."

The Basel Problem
Gottfried Wilhelm Leibniz (co-inventor of Calculus), the Bernouilli brothers and many other 1mathematicians were stumped by the problem of coming up with a closed form of the sum of the infinite series

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

Finding a closed form version of this sum (it was known to converge) and proving it was known as the Basel Problem.

Euler showed in 1735 that the solution was

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

Euler used two ideas:

1) The sum of the reciprocals of the roots of a polynomial equation written as $1 + c_1x + c_2x^2 + c_3x^3 + \dots + c_nx^n$ is equal to the negative coefficient of the linear term, i.e. $-c_1$. (This was a well-known result first proved by Rene Descartes) and

2) The "Maclaurin" series expansion of sin z= $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ..$

Howard Eves explains:

Then $\sin z = 0$ can (after dividing through by z) be considered as the infinite polynomial

$$1 - z^2/3! + z^4/5! - z^6/7! + \cdots = 0,$$

or, replacing z^2 by w, as the equation

$$1 - w/3! + w^2/5! - w^3/7! + \cdots = 0.$$

By the theory of equations, the sum of the reciprocals of the roots of this equation is the negative of the coefficient of the linear term—namely, $\frac{1}{6}$. Since the roots of the polynomial in z are $\pi, 2\pi, 3\pi, \dots$, it follows that the roots of the polynomial in w are $\pi^2, (2\pi)^2, (3\pi)^2, \dots$. Therefore

$$\tfrac{1}{6} = 1/\pi^2 + 1/(2\pi)^2 + 1/(3\pi)^2 + \cdots,$$

or

$$\pi^2/6 = 1/1^2 + 1/2^2 + 1/3^2 + \cdots.$$

EXERCISE

[5 EXTRA CREDIT POINTS] Use Euler's Method and the Maclaurin expansion for cos z to show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}$$