Senior Colloquium: *History of Mathematics*

Math 400 Spring 2020 | Fowler 310 T 1:30pm - 2:55pm

 | `http://sites.oxy.edu/ron/math/400/20/`

Homework #4

[5 points]

ASSIGNED: Tue Feb 18 2020 **DUE: Tue Feb 25 2020**

1. **Gaussian Quadrature.** Consider the function $f(x) = 1 - \frac{1}{x^2}$ on $1 \leq x \leq 3$. Our goal is to use 2-point Gaussian Quadrature to obtain approximate values of $\mathcal{I}$, the area between this curve, $x = 1$, $x = 3$ and the x-axis.

   (a) *1 point.* Evaluate $\mathcal{I} = \displaystyle\int_1^3 1 - \frac{1}{x^2}\, dx$ exactly.

   (b) *1 point.* Transform $\mathcal{I}$ into an integral on $-1 \leq x \leq 1$ so that you can (HINT: you should come up with a formula for transforming an integral on $[a, b]$ to one on $[-1, 1]$.)

   (c) *1 point.* Use the 2-point Gaussian Quadrature formula $f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$ to approximate $\mathcal{I}$. Do you expect the answer to be exact? Why or why not?

   (d) *2 points.* In general, the N-point Gaussian Quadrature rule will have an error formula that looks like

   $$\int_{-1}^{1} f(x)\, dx \approx \sum_{i=1}^{N} c_i f(x_i) + \frac{2^{2N+1}(N!)^4}{(2N+1)[(2N)!]^3} f^{2N}(\xi)$$

   where ξ is an unknown value in $[-1, 1]$ Use this formula to obtain an error bound for the approximation to $\mathcal{I}$ you found in (c). How does this formula explain why N-point Gaussian Quadrature will evaluate certain integrals **exactly**?