6 pts

MATH400 HW#2 SOLUTION

(a) $y^{n-1}\dfrac{dy}{dx} + a(x)y^n = f(x)$

$v = y^n$

$\dfrac{dv}{dx} = \dfrac{dv}{dy}\dfrac{dy}{dx} = ny^{n-1}\dfrac{dy}{dx} \Rightarrow \dfrac{dv}{dx}\cdot\dfrac{1}{n} = y^{n-1}\dfrac{dy}{dx}$

$\dfrac{1}{n}\dfrac{dv}{dx} + a(x)v = f(x) \Rightarrow \boxed{\dfrac{dv}{dx} + na(x)v = nf(x)}$

(b) Clairault's Equation is $y = xy' + f(y')$

Show that $y = Cx + f(C)$ is a solution to $y = xy' + f(y')$

Differentiate both sides $\quad y' = C$

$LHS = y = Cx + f(C)$

$RHS = xy' + f(y') = xC + f(C)$

$LHS = RHS$ ✓

(c) $\dfrac{dy}{dx} = p(x)y^2 + q(x)y + r(x)$

$y = v + \dfrac{1}{z}$ where $v = f(x)$ is a solution of the ODE

$\dfrac{dy}{dx} = \dfrac{dv}{dx} - \dfrac{1}{z^2}\dfrac{dz}{dx} = p(x)\left(v + \dfrac{1}{z}\right)^2 + q(x)\left(v + \dfrac{1}{z}\right) + r(x)$

$= p(x)\left(v^2 + \dfrac{2v}{z} + \dfrac{1}{z^2}\right) + q(x)v + \dfrac{q(x)}{z} + r(x)$

$= p(x)v^2 + q(x)v + r(x) + \dfrac{2v}{z}p(x) + \dfrac{q(x)}{z} + \dfrac{p(x)}{z^2} = \dfrac{dv}{dx} - \dfrac{1}{z^2}\dfrac{dz}{dx}$

$$2\frac{v}{z}p(x) + \frac{q(x)}{z} + \frac{p(x)}{z^2} = -\frac{1}{z^2}\frac{dz}{dx}$$

$$2vz\,p(x) + q(x)z + p(x) = -\frac{dz}{dx}$$

$$-z\left[2p(x)\cancel{v} + q(x)\right] - p(x) = \frac{dz}{dx}$$

Note $v = f(x)$ is known so this DE has the form

$$\frac{dz}{dx} = A(x)z + B(x) \quad \text{where } A(x) = -2p(x)f(x) - q(x)$$
$$B(x) = -p(x)$$

So ~~this~~ is a linear ODE in z