EC #3

The 2-point Gaussian Quadrature equations are

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(c_1) + w_2 f(c_2)$$

$f(x) = 1$:

$$\int_{-1}^{1} 1\,dx = w_1 + w_2$$

(1) $\boxed{2 = w_1 + w_2}$ ...

$f(x) = x$:

$$\int_{-1}^{1} x\,dx = w_1 c_1 + w_2 c_2$$

$$\frac{x^2}{2}\Big|_{-1}^{1} = \boxed{0 = w_1 c_1 + w_2 c_2}$$

$f(x) = x^2$:

$$\int_{-1}^{1} x^2\,dx = w_1 c_1^2 + w_2 c_2^2$$

$$= \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3} - -\frac{1}{3} = \boxed{\frac{2}{3} = w_1 c_1^2 + w_2 c_2^2}$$

$f(x) = x^3$:

$$\int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = \boxed{0 = w_1 c_1^3 + w_2 c_2^3}$$

Here's the system

$$w_1 + w_2 = 2 \Rightarrow w_2 = 2 - w_1$$

$$w_1 c_1 + w_2 c_2 = 0 \Rightarrow w_1 c_1 + (2 - w_1) c_2 = w_1 (c_1 - c_2) + 2c_2 = 0$$

$$w_1 c_1^2 + w_2 c_2^2 = \frac{2}{3} \Rightarrow w_1 c_1^2 + (2 - w_1) c_2^2 = \frac{2}{3}$$

$$w_1 c_1^3 + w_2 c_2^3 = 0$$

$$w_1 c_1^3 + (2 - w_1) c_2^3 = 0$$

$$w_1 c_1^2 - w_1 c_2^2 + 2c_2^2 = \frac{2}{3}$$

EC #3

2/2

$w_1(c_1 - c_2) + 2c_2 = 0 \Rightarrow w_1(c_1 - c_2) = -2c_2$

$w_1(c_1^2 - c_2^2) + 2c_2^2 = \frac{2}{3} \Rightarrow w_1(c_1 + c_2)(c_1 - c_2) + 2c_2^2 = \frac{2}{3}$

$w_1(c_1^3 - c_2^3) + 2c_2^3 = 0 \Rightarrow w_1(c_1 - c_2)(c_1^2 + c_1c_2 + c_2^2) + 2c_2^3 = 0$

$-2c_2(c_1 + c_2) + 2c_2^2 = \frac{2}{3} \Rightarrow -2c_2c_1 - 2c_2^2 + 2c_2^2 = \frac{2}{3}$

$c_1c_2 = -\frac{1}{3}$

$-2c_2(c_1^2 + c_1c_2 + c_2^2) + 2c_2^3 = 0$

$-2c_2c_1^2 + 2c_2^2c_1 - 2c_2^3 + 2c_2^3 = 0$

$-c_2c_1^2 = c_2^2c_1 \Rightarrow \frac{1}{3}c_1 = -\frac{1}{3}c_2 \Rightarrow c_1 = -c_2$

$c_2 = -c_1$

$-c_1^2 = -\frac{1}{3} \Rightarrow c_1^2 = \frac{1}{3} \Rightarrow c_1 = \frac{1}{\sqrt{3}} \Rightarrow c_2 = -\frac{1}{\sqrt{3}}$

$w_1 = \frac{-2c_2}{c_1 - c_2} = \frac{-2(-1/\sqrt{3})}{-2(-1/\sqrt{3})} = 1 \Rightarrow w_2 = 2 - w_1$

$w_2 = 2 - 1 = 1$

$w_1 = w_2 = 1 \quad c_1 = -c_2 = \frac{1}{\sqrt{3}}$