EC#2

D. Use the method of Calculus of Variations to prove that the shortest distance between two points is a straight line. In other words, considering $F(x,y,y') = \sqrt{1+y'^2}$ solve the Euler-Lagrange equation.

$$F(x,y,y') = \sqrt{1+(y')^2}$$

$$\frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial y'} = \frac{1}{2}(1+y'^2)^{-1/2} \cdot 2y'$$

$$= \frac{y'}{\sqrt{1+y'^2}}$$

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\frac{d}{dx}\left(\frac{y'}{\sqrt{1+y'^2}}\right) = 0$$

$$\Rightarrow \frac{y'}{\sqrt{1+y'^2}} = K$$

$$y' = K\sqrt{1+y'^2}$$

$$(y')^2 = K^2(1+(y')^2)$$

$$y'^2 - K^2y'^2 = K^2$$

$$y'^2(1-K^2) = K^2$$

$$y'^2 = \frac{K^2}{1-K^2}$$

$$y' = \sqrt{\frac{K^2}{1-K^2}}$$

$$y' = A$$

$$y = Ax+B$$

K, A & B are arbitrary constants

y(x) is linear.