EC #1

EXERCISE

[5 EXTRA CREDIT POINTS] Use Euler's Method and the Maclaurin expansion for cos z to show that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$$

$$\cos z = 1-\frac{z^2}{2!}+\frac{z^4}{4!}-\ldots = 0$$

Let $w=z^2 \Rightarrow \cos\sqrt{w}=0$

$$\sqrt{w}=\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots, \pm\frac{(2n-1)\pi}{2}$$

$$w=\left(\frac{\pi}{2}\right)^2, \frac{9\pi^2}{4}, \frac{25\pi^2}{4}, \ldots, \frac{(2n-1)^2\pi^2}{4}$$

$$1-\frac{w}{2!}+\frac{w^2}{4!}-\frac{w^3}{6!}+\ldots = 0$$

By Descartes Rule that sum of the reciprocal of the roots of a polynomial equals the negative ~~reciprocal~~ coefficient of the linear term

$$\frac{1}{\left(\frac{\pi}{2}\right)^2}+\frac{1}{\left(\frac{9\pi^2}{4}\right)}+\ldots+\frac{1}{\frac{(2n-1)^2\pi^2}{4}}=\frac{1}{2!}$$

$$\sum_{n=1}^{\infty}\frac{1}{\frac{(2n-1)^2\pi^2}{4}}=\frac{1}{2!}$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{4}\cdot\frac{1}{2!}=\frac{\pi^2}{8}$$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$$