History of Mathematics

Math 395 Spring 2010 — Fowler 310 MWF 10:30am - 11:25am

 — http://faculty.oxy.edu/ron/math/395/10/

QUIZ #3: 04/16/2010

NAME: Key **TOTAL**_____/30

MATH PART I:

(5 points) Verify a result of Descartes that the sum of the reciprocal of the roots of an arbitrary quadratic equation written $1+c_1x+c_2x^2=0$ is equal to $-c_1$. [HINT: Recall that the quadratic equation in the form $(x-a)(x-b)=0$ has roots at *a* and *b*.]

$(x-a)(x-b) = x^2 - x(a+b) + ab = 0$

$\frac{x^2}{ab} - x\left(\frac{a+b}{ab}\right) + 1 = 0 \Rightarrow c_2x^2 + c_1x + 1 = 0$

$c_2 = \frac{1}{ab} \quad c_1 = -\frac{a+b}{ab} = -\left(\frac{1}{b}+\frac{1}{a}\right)$

$-c_1 = \frac{1}{a}+\frac{1}{b}$

= Sum of ~~root~~ reciprocal of roots (cool!!)

MATH PART II:

(10 points) Consider $y=\sqrt{x}$. Use either Newton's method of fluents and fluxions **or** Leibniz' method of differentials to show that $\dot{y}=\frac{\dot{x}}{2\sqrt{x}}$ or $dy=\frac{dx}{2\sqrt{x}}$. [For half-credit you can use the modern limit definition of the derivative to differentiate the given function.]

Show all your work!

Leibniz

$y = \sqrt{x}$

$dy = \sqrt{x+dx} - \sqrt{x}$

$= \left(\sqrt{x+dx}-\sqrt{x}\right)\frac{\sqrt{x+dx}+\sqrt{x}}{\sqrt{x+dx}+\sqrt{x}}$

$= \frac{x+dx-x}{\sqrt{x+dx}+\sqrt{x}}$

$= \frac{dx}{\sqrt{x+dx}+\sqrt{x}}$ since $dx \ll x$

$= \frac{dx}{\sqrt{x}+\sqrt{x}}$

$= \frac{dx}{2\sqrt{x}}$

Newton

$y+o\dot{y} = \sqrt{x+o\dot{x}}$

$o\dot{y} = \sqrt{x+o\dot{x}} - \sqrt{x}$

$o\dot{y} = \sqrt{x\left(1+\frac{o\dot{x}}{x}\right)} - \sqrt{x}$

$o\dot{y} = \sqrt{x}\left(1+\frac{o\dot{x}}{x}\right)^{1/2} - \sqrt{x}$

$o\dot{y} = \sqrt{x}\left[1+\frac{1}{2}\frac{o\dot{x}}{x} - \frac{1}{8}\left(\frac{o\dot{x}}{x}\right)^2 + \ldots\right] - \sqrt{x}$

$o\dot{y} = \frac{1}{2}\sqrt{x}\,\frac{o\dot{x}}{x} - \frac{1}{8}\frac{o^2\dot{x}^2}{x^2}\sqrt{x} + \ldots$

divide by o

$\dot{y} = \frac{1\sqrt{x}}{2}\frac{\dot{x}}{x} - \frac{1}{8}\frac{o\dot{x}^2}{x^2}\sqrt{x} + o^2 + \ldots$

Terms with o small

$\dot{y} = \frac{1}{2}\frac{\dot{x}}{\sqrt{x}}$

HISTORY PART I: LONG-ANSWER QUESTION (5 points).
(5 points) If Newton or Leibniz had never been born, who do you think would have invented Calculus? **WRITE LEGIBLY and provide a full paragraph (i.e. multiple sentences) to support your answer!**

Reasonable answers Barrow, John or Jakob Bernouilli, Euler, McLaurin

Need major support: Fermat, Pascal, Descartes

HISTORY PART II: SHORT-ANSWER QUESTIONS (5 points)
Write down whether the following sentences are either TRUE or FALSE.

A. F Newton and Leibniz were uninterested in who received final credit for the invention of Calculus.

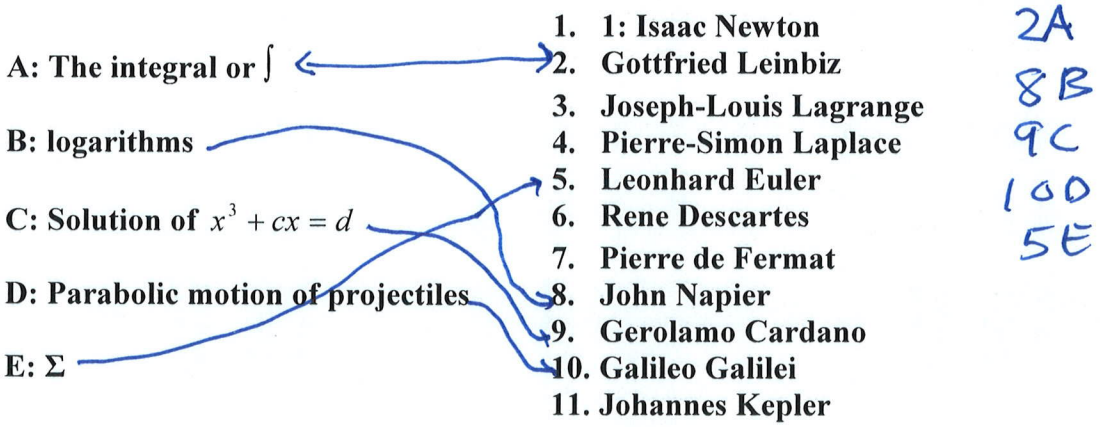
B. T Fermat was able to build upon the work of Viète to solve algebraic problems.

C. F Kepler was always a stickler for matching theory to experimental observations.

D. F Galileo was able to build upon the work of the Bernouilli brothers to solve the catenary problem.

E. F Descartes believed geometry and algebra were two distinct, unrelated fields of study.

HISTORY PART III: MATCH QUESTION (5 points)
Match the concept, symbol or equation with the name of the one Mathematician most closely associated with it.



2A
8B
9C
10D
5E