Senior Colloquium: *History of Mathematics*

Math 400 Fall 2019 — Library 355 T 3:05pm - 4:30pm

 — http://sites.oxy.edu/ron/math/400/19/

Worksheet 11

TITLE Non-Western Perspectives, Part 2: Mathematics from China and Africa
CURRENT READING: Katz §10 (pp 324-363); Boyer & Merzbach, §12-13 (pp 223-281)
Gerdes (*Historia Mathematica*, Vol. 21, 345-376)

SUMMARY
We continue looking at mathematics knowledge in non-European sites around the world

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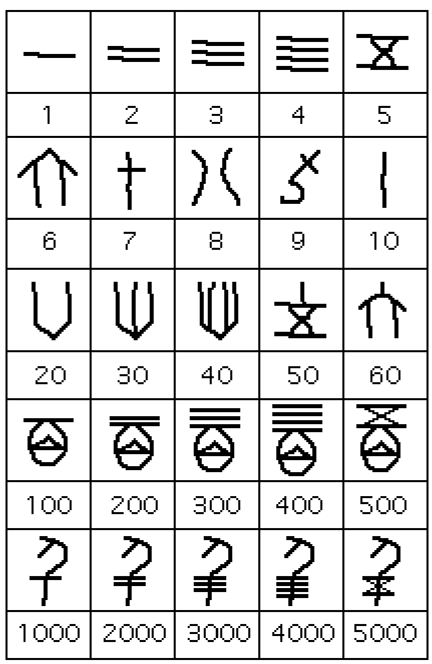
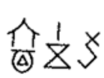
NEXT: Early 17th Century Stars and the Prelude to Calculus: Mersenne, Fermat, Pascal and Galileo
NEXT READING: Katz §14-15 (pp 467-541) and Boyer & Merzbach, §15 (pp 300-348)

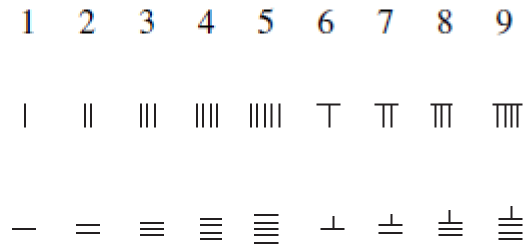
Number System
The Chinese number system was decimal, similar to the Egyptian one, with many different symbols used. However, in the Chinese system there were separate symbols for the first 9 digits AND some multiples of ten. (Recall that the Egyptian Hieroglyphic number system just had independent symbols for powers of 10.)



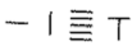
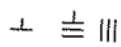
EXAMPLE

What number does  represent? How would you represent the number 3282?

Katz reports that the Chinese apparently also represented numbers using small bamboo rods, called counting rods in a decimal place system. They represented negative numbers by using different colors. When a particular place was empty it would be denoted by a small dot (representing zero).



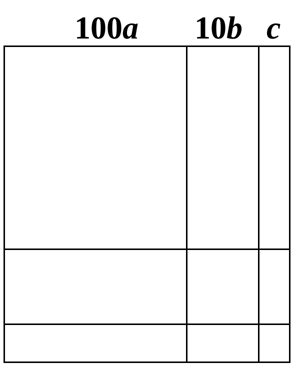
Exercise

 represents 1156 while 

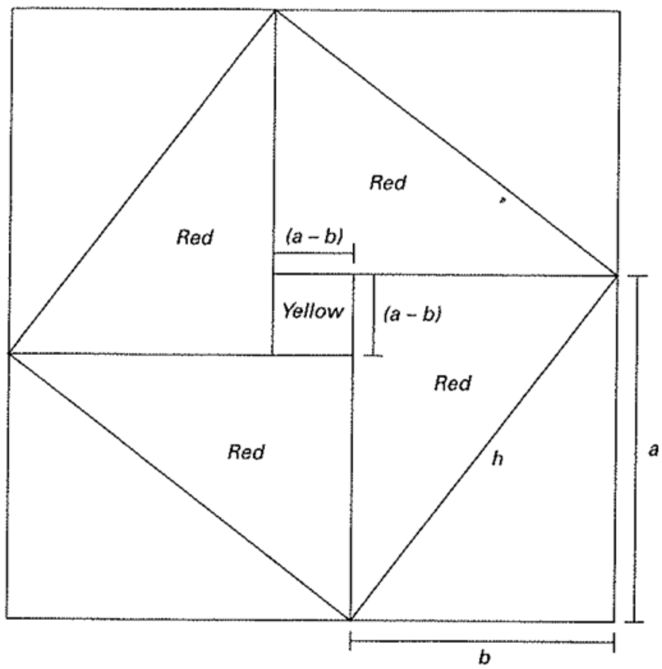
Nine Chapters on the Mathematical Art (Jiuzhang suanshu)
The most famous of Ancient Chinese mathematical works is *Jiuzhang suanshu* which is primarily know from the version commented on by Liu Hui in the Third Century CE.

GroupWork

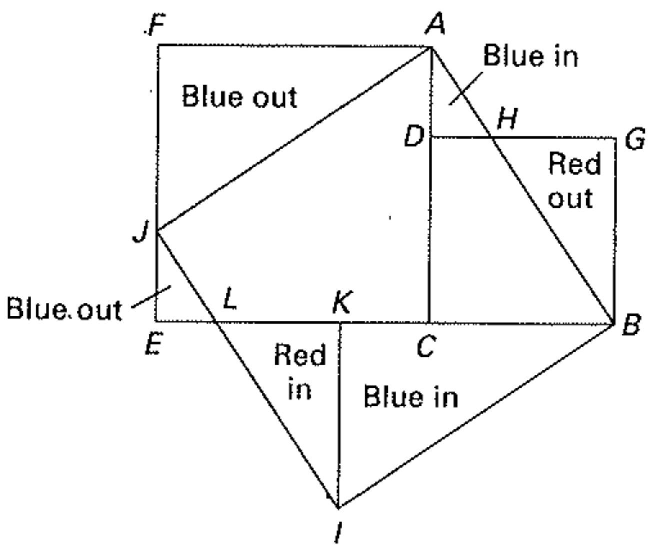
Let's replicate the Chinese square root algorithm to evaluate "the side of a square of area 55,225"



The *gougu* Rule (Pythagoras' Theorem)
Katz gives two different proofs of Pythagoras theorem, one due to Zhao Shuang in *Arithmetic Classic of the Gnomon*



And Hui's proof



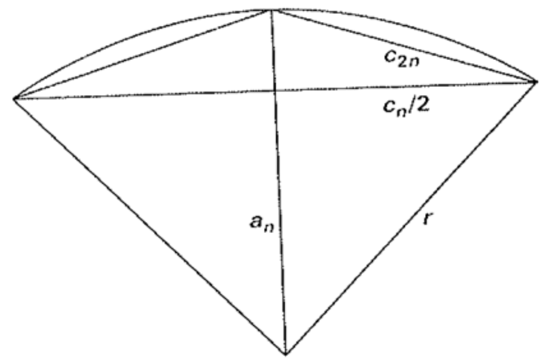
Standards of Proof
What can we say about the standard of proof used by Chinese mathematicians as compared to the Greeks and modern standards?

Calculation of Pi

$$a_n = \sqrt{r^2 - \left(\frac{c_n}{2}\right)^2} \quad \text{and} \quad c_{2n} = \sqrt{\left(\frac{c_n}{2}\right)^2 + (r - a_n)^2}.$$

Then

$$S_{2n} = 2n\frac{1}{2}\frac{c_n}{2}r = \frac{1}{2}nrc_n.$$



By computing the area of a regular-sided *n*-gon, S_n, and the corresponding *2n*-gon, Liu was able to approximate π by using r=10 and n=96 to obtain $\pi \sim 3.141024$.
Later, Zu Chingzhi (c. 429-500) continued the calculations using n=24576 to obtain $\pi \sim 3.1415926$

Magic Square
The earliest known magic square was found by the Chinese (Struik, *On Ancient Chinese Mathematics*). RECALL: A magic square is a 3x3 matrix where sums along the rows and columns and major diagonals are all equal (i.e. 15 in the square below).

4	**9**	**2**
3	**5**	**7**
8	**1**	**6**

The search for other magic squares apparently lead to the solution of linear systems of equations and a method very similar to Gaussian elimination.

$$3x+2y+z=39$$
$$2x+3y+z=34$$
$$x+2y+3z=26$$

becomes

1 2 3
2 3 2
3 1 1
26 34 39

0 0 3
0 5 2
36 1 1
99 24 39

Which corresponds to *3x+2y+z=3, 5y+z=24, 36z=99*. **How does this method differ from Gaussian elimination?**

Simultaneous linear congruences

In *Mathematical Classic of Master Sun* (*Sunzi suanjing*) from 300 CE the following problem appears:

> ***We have things of which we do not know the number; if we count them by threes, the remainder is 2; if we count them by fives, the remainder is 3; if we count them by sevens, the remainder is 2. How many things are there?***

In modern notation, this becomes a problem of simultaneous linear congruences:

Find N, such that

$$N \equiv 2 \pmod 3 \quad N \equiv 3 \pmod 5 \quad N \equiv 2 \pmod 7 \qquad (1)$$

The answer is N=23.

Katz reports Sun Zi's solution:

> "If you count by threes and have the remainder 2, put 140. If you count by fives and have the remainder 3, put 63. If you count by sevens and have the remainder 2, put 30. Add these numbers and you get 233. From this subtract 210 and you get 23."

It turns out that

$70 \equiv 1 \pmod 3) \equiv 0 \pmod 5) \equiv 0 \pmod 7)$
$21 \equiv 1 \pmod 5) \equiv 0 \pmod 3) \equiv 0 \pmod 7)$
$15 \equiv 1 \pmod 7) \equiv 0 \pmod 3) \equiv 0 \pmod 2)$

So, if you want to find N which satisfies all three equations in (1) simultaneously it can be computed as

$$N = 70 \text{ x } 2 + 21 \text{ x } 3 + 15 \text{ x } 2 = 140 + 63 + 30 = 233 = 23 \pmod{105}$$

The modern Chinese Remainder Theorem is the generalized version of the Sun Zi problem.

Theorem: Let p, q be coprime. Then the system of equations

$$x = a \pmod p$$

$$x = b \pmod q$$

has a unique solution for x modulo pq.

Exercise

Show that the solution to $N \equiv 2 \pmod 5$ and $N \equiv 3 \pmod 7$ is N = 17 (mod 35)

The Hundred Fowls Problem

In *Mathematical Classic of Zhang Quijian* from the 5th century CE the hundred flows problem appears:

“A rooster is worth 5 coins, a hen 3 coins, and 3 chicks 1 coin. With 100 coins we buy 100 of the fowls. How many roosters, hens and chicks are there?”

This corresponds to the system of two equations in three unknowns:

$$5x+3y+\frac{1}{3}z=100$$
$$x+y+z=100$$

Zhang gave three answers: “4 roosters, 18 hens, 78 chicks; 8 roosters, 11 hens, 81 chicks; 12 roosters, 4 hens, 84 chicks.”

GroupWork

Show that the general solution of this system is $x=-100+4t$, $y=200-7t$, $z=3t$.