Worksheet 3: **Tuesday September 10**

TITLE Introduction to Ancient Greek Mathematicians and Greek Mathematics
CURRENT READING: Katz, §2 (pp 32-47); Boyer & Merzbach, (pp 40-56, 74-80)

Next Time: More discussion of mathematical results of the Greeks (logic, geometry, etc)

SUMMARY
We begin to look at the contributions of Ancient Greece to mathematics.

Greek Numbers
The Greeks used a cipher system where each letter in the Greek alphabet represented a particular number and then some letters that were no longer in use were added for certain numbers (6, 90, 900). To represent thousands a diacritical mark like ` was placed to the left of the first nine numerals. So, `βιθ was equal to 2019. You could get even larger numbers by using *M* (representing "myriads") and a superscript. So, M^{δ} = 40,000 and $M^{`\zeta\rho o\varepsilon}$ = 71,750,000.

Representation of a number system used by the Greeks as early as the sixth century BCE.

Letter	Value	Letter	Value	Letter	Value
α	1	ι	10	ρ	100
β	2	κ	20	σ	200
γ	3	λ	30	τ	300
δ	4	μ	40	υ	400
ϵ	5	ν	50	ϕ	500
ϛ	6	ξ	60	χ	600
ζ	7	o	70	ψ	700
η	8	π	80	ω	800
θ	9	ϙ	90	ϡ	900

The diacritical mark to the right represented fractions:

$$\beta' = \frac{1}{2} \quad and \quad \mu\beta' = \frac{1}{42}$$

(Note that the last expression could also mean 40½). They could write more complex fractions using an overbar. For example,

$$\overline{\nu\alpha}\ \pi\delta = \frac{51}{84}$$

EXAMPLE

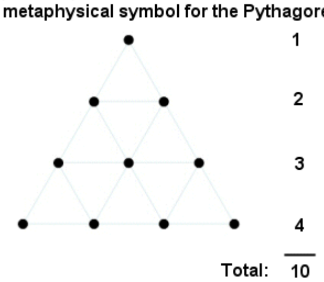
Write 42 in Greek numbers. What does χξς represent? And $M^{\alpha\kappa}\sigma\pi\zeta$?

The Phythagorean School

The motto of the Pythagorean school was “All is number.” They imbued numbers with mysticism. They ascribed very specific properties to specific numbers:

- **One**: the number of reason
- **Two**: the number of opinion, the first even of female number
- **Three**: the number of harmony, the first “true” male number
- **Four**: the number of justice or retribution
- **Five**: the number of marriage (i.e. 3+2)
- **Six**: the number of creation
- **Seven**: the number of the planets (or “wandering stars”)
- **Eight**:
- **Nine**:
- **Ten**: “tetractys” the holiest number; the number of the universe

The tetractys, an equilateral triangular figure consisting of 10 points arranged in four rows of 1, 2, 3 and 4, was both a mathematical idea and a metaphysical symbol for the Pythagoreans.



Classifying numbers: Amicable (or Friendly), Perfect, Deficient, and Abundant

The Pythagoreans distinguished between **arithmetic**, the abstract relationship between numbers, and **logistic**, the practical art of computing with numbers.

Amicable or friendly numbers

Two number are said to be amicable or friendly if each is the sum of the proper divisors of the other. EXAMPLE: 284 and 220 are friendly numbers.

EXAMPLE: 284 (1,2,4,71,142) and 220 (1,2,4,5,10,11,20,22,44,55,110) are amicable.

Perfect, deficient and abundant numbers.

A number is said to be perfect if it is the sum of its proper divisors. The number is deficient if it is greater than the sum of its proper divisors and abundant if it is less than the sum of its proper divisors.

EXAMPLE: **6 is a perfect number, 8 is a deficient number, 12 is an abundant number.**

Proper divisor

The proper divisors of a positive integer *N* are all the positive whole numbers that divide *N* exactly except for N itself. NOTE: 1 is always a proper divisor of *N*. (So is *N*). Another (archaic) term for proper divisor is *aliquot part*.

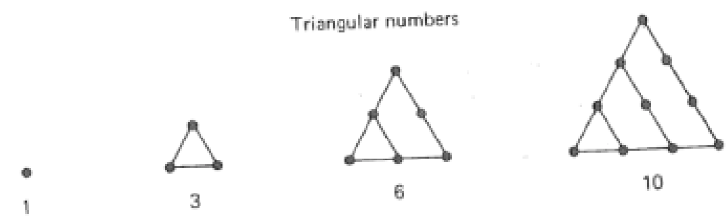
Prime number

A number is said to be prime if it is a positive integer greater than 1 without having any positive whole number divisors except for itself.

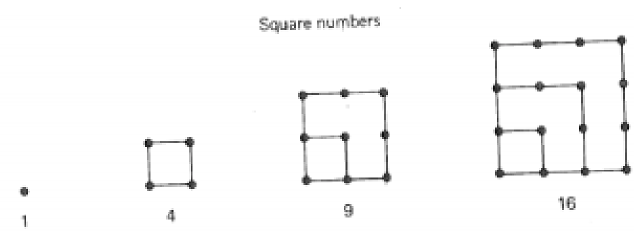
Figurate Numbers

The Pythagoreans originated the concept of the "figurate" numbers. These are sequences of numbers associated with placing a dot in a geometric pattern and then increasing the number of dots to maintain the pattern. This represents a link between geometry and arithmetic that the Pythagoreans would have greatly appreciated.

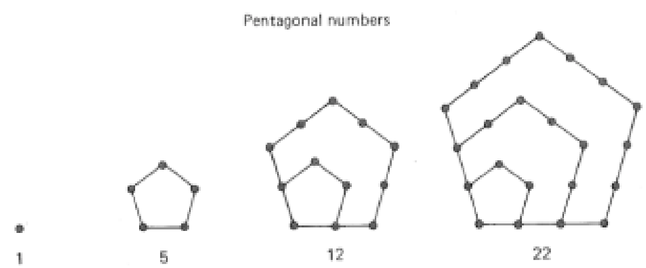
Triangular Numbers



Square Numbers



Pentagonal Numbers



GROUPWORK

Extend the list of triangular, square and pentagonal numbers. Do you notice any patterns? Can you find a formula for the n^{th} number in each of these sequences?

GroupWork

Describe the main reason why we still know the following names below and summarize their contributions to mathematics.

Thales (c. 624-547 BCE)

Pythagoras (c. 572-497 BCE)

Plato (429-347 BCE)

Aristotle (384-322 BCE)

Zeno (c. 495-c. 430 BCE)