# Senior Colloquium: *History of Mathematics*

Math 400 Fall 2019 | Library 355 T 3:05pm - 4:30pm



http://sites.oxy.edu/ron/math/400/19/

*Worksheet 1*: **Tuesday August 27**

**TITLE** Introduction to History of Mathematics: Origins, Egypt
**CURRENT READING** Katz, 1-14; Boyer 1-12

**Homework Assignments due Friday August 30 5PM**
HW #0: Automathography. E-mail me a three-paragraph (at least 250 words) "automathography" of yourself. Automathography is a neologism composed of the word autobiography and math, and it is basically a biography of your mathematical self, written by you. Tell me about the courses you have taken, what your favorites were, what you find hardest. Explain what your mathematical interests are, and what you plan to do after graduation. Reveal why you signed up for this course and what you expect to get out of it. If your aims for taking this course are different from the stated goals in the syllabus, please let me know. If you have any anxieties about this course, or any special problems or needs, let me know. You are encouraged to be creative in your response; don't be pedantic and just answer the questions asked above; include whatever you wish.

**SUMMARY**
In today's class we shall go through an overview of the class, discuss the syllabus, and begin our discussion of the origin of mathematics

## 1. Origin of Mathematics

The origins of mathematics accompanied the evolution of social systems. Many, many social needs require

- counting,
- calculations,
- measurement

EXAMPLE

- The worth of a herdsman cannot be known unless some basics facts of counting are known.
- A temple cannot be built unless certain facts about triangles, squares, and volumes are known.
- An inheritance cannot be distributed unless certain facts about division (fractions) are known.

From practical needs such as these, mathematics was born.

## Cardinals

One view is that the core of early mathematics is based upon two simple questions.

- **How many?**
- **How much?**

This is the *cardinal* number viewpoint.

## Ordinals

Another view is that mathematics may have an even earlier basis on ordinals used perhaps for rituals in religious practices or simply the pecking order for eating the fresh game. This is the *ordinal* viewpoint.

Such basic questions are thus:

- Who is first, etc?
- What comes first, etc?

We will take the cardinal numbers viewpoint in our discussion.

# HOW MANY?

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Systems of enumeration.

    Primitive:
        notches, sticks, stones
    Egyptians:
        symbols for 1, 10, 100, 1,000, ... 1,0000,000.
    Babylonians:
        two symbols only--cuneiform
    Greeks:
        alphabetical denotations, plus special symbols
    Roman:
        Roman numerals, I,V,X,L,C,D,M..
    Arabs:
        Ten special symbols for numbers.
    Modern:
        Ten special symbols for numbers.

Methods of ciphering.

    Devices:
        Abacus, counting boards.
    Symbolic:
        Arithmetic.
```

**HOW MANY?**

Bases for numbering systems

- binary – early
- ternary – early
- quinary – early
- decimal
- vigesimal
- sexigesimal
- combinations of several

## HOW MUCH?

When counting or asking how many, we can limit discussions to whole positive integers. When asking how much, integers no longer suffice.

Examples:

Given 17 seedlings, how can they be planted in five rows?
Given 20 talons of gold, how can they be distribution to three persons?
Given 12 pounds of salt, how can it be divided into five equal containers?

When asking how much we are led directly to the need of fractions.

HOW MUCH?

Another how much question is connected with measurement.

- Construction. To build granaries, or ovens to bake bread, or pyramids, or temples we need formulas for quantity, or area or volume.
- Planting. To divide arable plots we need formulas for plane area and those for seasons.
- Astronomy. To study the motions of stars we need angular and temporal measurement.
- Taxes and commerce. To properly assess taxes, we need ways to compute percentages (fractions).

**Egyptian Mathematics**

. They used base 10 (decimal).
. Their number system was NOT positional.
. They only allowed unit fractions except for 2/3 and 3/4.
. They could solve 2 equations in 2 unknowns
. They well understood the distribution properties of multiplication and addition.
. They were well-versed in arithmetic operations with fractions
. Possessed two number systems: hieroglyphic and hieratic

**Egyptian Hieroglyphic Numbers**

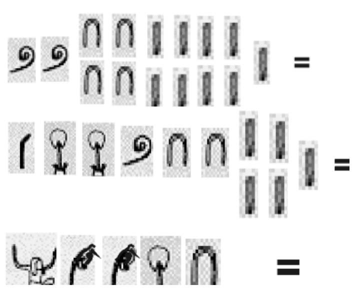
| | | |
|---|---|---|
| = 1 | = 1,000 | = 1,000,000 |
| = 10 | = 10,000 | |
| = 100 | = 100,000 | |

1 = vertical stroke
10 = heal bone
100 = a snare

1,000 = lotus flower
10,000 = a bent finger
100,000 = a burbot fish

1,000,000 = a kneeling figure

EXAMPLE What numbers do the following hieroglyphics represent?



**Addition** is executed by grouping

**Exercise**

What sum is represented by the figures above?

**Multiplication** and **Division** are both binary

Example: Multiply: $47 \times 24$

| | | | |
|---|---|---|---|
| 47 | $\times$ | 24 | |
| 47 | | 1 | doubling process |
| 94 | | 2 | |
| 188 | | 4 | |
| 376 | | 8 | * |
| 752 | | 16 | * |

Selecting 8 and 16 (i.e. $8 + 16 = 24$), we have

$$\begin{aligned} 24 &= 16 + 8 \\ 47 \times 24 &= 47 \times (16 + 8) \\ &= 752 + 376 \\ &= 1128 \end{aligned}$$

Example: 329 ÷ 12

| | | | | |
|---|---|---|---|---|
| 329 | ÷ | 12 | | |
| 12 | | 1 | doubling | 329 |
| 24 | | 2 | | -192 |
| 48 | | 4 | | 137 |
| 96 | | 8 | | -96 |
| 192 | | 16 | | 41 |
| 384 | | 32 | | -24 |
| | | | | 17 |
| | | | | -12 |
| | | | | 5 |

Now

$$\begin{aligned} 329 &= 16 \times 12 + 8 \times 12 + 2 \times 12 + 1 \times 12 + 5 \\ &= (16 + 8 + 2 + 1) \times 12 + 5 \end{aligned}$$

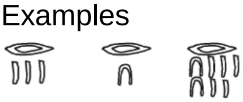
So,

$$329 \div 12 = 27\,\frac{5}{12} = 27 + \frac{1}{3} + \frac{1}{12}.$$

**Fractions**

Fractions were written with a symbol, called the **horus-eye**, in the numerator over the hieroglyphics for the number in the denominator. In hieratic, a dot was placed over the hieratic symbol for the number in the denominator.

Examples



1/3 1/10 1/25

There were special symbols for ½, 2/3 and 3/4

1/2 2/3 3/4

The Katz book uses $\bar{n}$ to represent 1/n and $\bar{\bar{3}}$ to represent 2/3

**Babylonian Mathematics**
. They used no zero.
. More general fractions, though not all fractions, were admitted.
. They could extract square roots.
. They could solve linear systems.
. They worked with Pythagorean triples.
. They solved cubic equations with the help of tables.
. They studied circular measurement.
. Their geometry was sometimes incorrect.

For enumeration the Babylonians used symbols for 1, 10, 60, 600, 3,600, 36,000, and 216,000. Below are four of the symbols.

Cuneiform numerals

They did arithmetic in base 60, ***sexagesimal***
For our purposes we will use just the first two symbols

$$\vee \;=\; 1 \qquad\qquad \prec \;=\; 10$$

All numbers will be formed from these.

Example:

$$\begin{matrix}\prec\prec & \vee\,\vee\,\vee \\ \prec\prec\prec & \vee\,\vee\,\vee\vee\end{matrix} = 57$$

Note the notation was **positional** and **sexagesimal**:

$\prec\prec\;\;\prec\prec = 20 \cdot 60 + 20$ and $\vee\,\vee\;\;\vee\vee\;\;\prec\;\vee$ $=2\cdot 60^2+2\cdot 60+11=7{,}331$

**Notation**
We will write these numbers as **20,20** and **2,2,11**

**In general**
It should be noted that Babylonians did not use a zero but just left a space if a number was missing a particular power (Katz, 12).

**Why Base 60?**
Many different reasons were given for this choice, some are:
1. The number of days, 360, in a year gave rise to the subdivision of the circle into 360 degrees, and that the chord of one sixth of a circle is equal to the radius gave rise to a natural division of the circle into six equal parts. This in turn made 60 a natural unit of counting.
2. The Babylonians used a 12 hour clock, with 60 minute hours.
3. The base 60 provided a convenient way to express fractions from a variety of systems as may be needed in conversion of weights and measures.
4. The number 60 is the product of the number of planets (5 known at the time) by the number of months in the year, 12.
5. The combination of the duodecimal system (base 12) and the base 10 system leads naturally to a base 60 system.

## Babylonian Computations

### Addition

**Exercise**

Show that 23,37+41,32=1,5,9

### Multiplication

**Exercise**

Show that 34 X 9 =5,6

### Division and Fractions

The Babylonians only dealt with **regular** sexagesimal numbers, that is numbers whose reciprocal is a terminating sexagesimal fraction (Katz, 14).

**EXAMPLE**

$$\frac{1}{6} = \frac{10}{60} = ; \prec$$

$$\frac{1}{9} = ; \begin{matrix} \vee\vee\vee \\ \vee\vee\vee \end{matrix} , \begin{matrix} \prec\prec \\ \prec\prec \end{matrix}$$

**Exercise**

How would you compute 1/8 as a sexigesimal fraction?

These reciprocals were often calculated with the assistance of tables.

A table of all products equal to sixty has been found.

| | | | |
|---|---|---|---|
| 2 | 30 | 16 | 3, 45 |
| 3 | 20 | 18 | 3,20 |
| 4 | 15 | 20 | 3 |
| 5 | 12 | 24 | 2,30 |
| 6 | 10 | 25 | 2,25 |
| 8 | 7,30 | 27 | 2,13,20 |
| 9 | 6,40 | 30 | 2 |
| 10 | 6 | 32 | 1;52,30 |
| 12 | 5 | 36 | 1,40 |
| 15 | 4 | 40 | 1,30 |

From the table we can see that

$$8 \times 7;30 = 8 \times (7 + \frac{30}{60}) = 60$$

It can also be used to compute reciprocals

$$\frac{1}{8} = 0;7,30 = \frac{7}{60} + \frac{30}{60^2}$$

**Exercise**

What would an equivalent table look like for our decimal system?

(HINT: You only keep fractions which have terminating decimal representations!)