An Economic Model of Discrimination

Goals for Today

- Explore the long-term effects of prejudiced beliefs on competitive market outcomes using an economic model
- Think through how elements of the model affect conclusions, whether model is close enough to reality.
- Mathematical Concepts:
  - Expected Value
  - Bayes Theorem
- Economics Concepts:
  - Statistical Discrimination
  - Nash Equilibrium

The Real-World Problem

- Racial Discrimination is prevalent in labor markets, housing markets, and many other economic interactions.
  - Ex: Bertrand 2004.
- Discrimination is part of a dynamic system: expectations of discrimination change worker’s behavior and incentives, change behavior of other actors in society.
- Want to know: what sorts of efforts/interventions are most likely to lead to good outcomes across the whole system?

How do Economists Use Models?

- This model is a storytelling tool:
    - Goal is to show that a certain outcome *might* happen, and make it clear *why* it might happen.

- Not trying to capture all nuances of the real world, be measurable with real data, or make specific empirical predictions.

Simple Model: Statistical Discrimination

- Firm’s motive: Maximize profit. Doesn’t *care* about race directly.
- Race is *correlated* with some characteristic that affects worker productivity
- If characteristic not observed, race will predict productivity.

Example: Yacht Salesman

- Easier to Sell Yachts if you have a rich friend
- White employees more likely to have rich friend than Black employees
- Salesepeople with rich friends sell $160,000/year on average
- Without rich friends: $40,000/year on average:

	Frac. Rich Friend $160,000/year	Frac. No Rich Friend $40,000/year
White	¾	¼
Black	¼	¾

Example: Yacht Salesman

- Firm cares about Expected Value of Yacht Sale:
- $E(X) = \sum_{x \in X} P(x) * x$

Expected Sales:

E(S) = P(R=1)*160,000 + P(R=0)*40,000

	Frac. Rich Friend $150,000/year	Frac. No Rich Friend $50,000/year	E(S) = P(R=1)*150,000 + P(R=0)*50,000
White	¾	¼	¾*160,000 + ¼*40,000 = 130,000
Black	¼	¾	¼*160,000 + ¾*40,000 = 70,000

Consequence

- Firm trying to maximize profits will pay $130,000 to White workers, $70,000 to Black workers.

- Firms that *don't* discriminate will lose money

- Statistical Discrimination IS Discrimination.

Coate and Lourey Model:

What is the consequence of statistical discrimination on worker decisions?

Setup

- Two jobs: Laborers and Engineers
  - Engineering pays more, requires skills.
- Workers want to maximize income:
  - Choose whether to invest in skills
- Firms want to maximize profits:
  - Choose who to hire as engineers
- Firms can’t directly see who is skilled. Only observe results of a flawed test.

Firm’s problem:

Revenue to Firm	Laborer Wage=$40,000	Engineer Wage=$100,000
Skilled	$40,000	$200,000
Unskilled	$40,000	$0

Goal: Maximize Profits

Hire workers as engineers if expected profit is positive

- $E(\pi) = \pi(S = 1) * P(S = 1) + \pi(S = 0) * P(S = 0)$

Call P the probability that a worker is skilled: what is the minimum P at which it’s worth it to hire a worker as an engineer?

Firm’s problem:

Revenue to Firm	Laborer Wage=$40,000	Engineer Wage=$100,000
Skilled	$40,000	$200,000
Unskilled	$40,000	$0

Min P to hire an engineer:

$$E(\pi) = \pi(S=1) * P + \pi(S=0) * (1-P) \geq 0$$
$$E(\pi) = (200k - 100k) * P + (0 - 100k) * (1-P) > 0$$
$$100k * P + 100k * P \ - 100k \geq 0$$
$$P \geq \frac{1}{2}$$

But Firms don’t know P

- Firms know whether workers passed a test (Z=1)

- 100% of skilled workers pass
- 1/3 of unskilled workers pass

- Return to this in a moment.

The Workers

- Goal: Maximize take-home earnings:
- Choose: Pay $20,000 to become skilled?

If all workers hired as engineers who pass test:

E(Earn|S=1) = Earn(Z=1)*P(Z=1|S=1) + Earn(Z=0)*P(Z=0|S=1) - $20,000

E(Earn|S=1) = $100,000*1 + $40,000*0 -$20,000 = $80,000

E(Earn|S=0) = Earn(Z=1)*P(Z=1|S=0) + Earn(Z=0)*P(Z=0|S=0)

E(Earn|S=0) = $100,000*1/3 + $40,000*2/3 = 60,000

So: If workers who pass test are hired as engineers, all workers will become skilled.

The Firm

- Is it worth it to hire people who pass the test as engineers?

- Two ways to pass the test:
  - A) Be skilled
  - B) Be unskilled but pass by luck

- What is probability that someone who passed the test is skilled?

$$P(S = 1|Z = 1) = \frac{Skilled\ test\ passers}{Skilled\ test\ passers + Unskilled\ test\ passers}$$

Bayes Theorem

- General solution to this problem: how do we find P(S=1|Z=1)—probability that a test passer is skilled?

$$P(S = 1|Z = 1) = \frac{Skilled\ test\ passers}{Skilled\ test\ passers + Unskilled\ test\ passers}$$

Skilled test passers = P(Z=1|S=1)*P(S=1)

Unskilled test passers = P(Z=1|S=0)*P(S=0)

Bayes Cont.

$$P(S=1|Z=1) = \frac{Skilled\ test\ passers}{Skilled\ test\ passers + Unskilled\ test\ passers}$$

$$P(S=1|Z=1) = \frac{P(Z=1|S=1) * P(S=1)}{P(Z=1|S=1) * P(S=1) + P(Z=1|S=0) * P(S=0)}$$

- In other words, likelihood that someone who passed test is skilled depends on how many people you think are skilled overall (P(S=1)).

Bayes In General

$$P(X = x|Y = y) = \frac{P(X = x\ AND\ Y = y)}{P(y = Y)}$$

$$P(X = x|Y = y) = \frac{P(Y = y|X = x) * P(X = x)}{\sum_{x' \in X} P(X = x'|Y = y) * P(X = x')}$$

Hiring Rule if firms think workers are skilled

If firms think most workers skilled, firms will hire all test-passers. Suppose P(S=1)=0.9:

$$P(S=1|Z=1) = \frac{P(Z=1|S=1) * P(S=1)}{P(Z=1|S=1) * P(S=1) + P(Z=1|S=0) * P(S=0)}$$

$$P(S=1|Z=1) = \frac{1 * 0.9}{1 * 0.9 + 0.333 * 0.1} = \frac{0.9}{0.933} = 0.97$$

So: firms hire test-passers, because P(S=1|Z=1)>1/2.

When firms hire test passers, all workers become skilled.

Thus, all workers become skilled, get hired as engineers, everything is great.

Hiring Rule if firms think workers aren't skilled

If firms think most workers are not skilled, firms will hire no test-passers. Suppose P(S=1)=0.2:

$$P(S = 1|Z = 1) = \frac{P(Z = 1|S = 1) * P(S = 1)}{P(Z = 1|S = 1) * P(S = 1) + P(Z = 1|S = 0) * P(S = 0)}$$

$$P(S = 1|Z = 1) = \frac{1 * 0.2}{1 * 0.2 + 0.333 * 0.8} = \frac{0.2}{0.4222} = 0.47$$

So: firms hire NO test-passers, because P(S=1|Z=1)<1/2.

When firms hire no test passers, no workers become skilled.

Thus, no workers become skilled, firms conclude that no workers are skilled, and continue not to hire engineers.

Nash Equilibrium

	Firms Hire Test Passers	Firms Don't Hire Test passers
Workers Get Skilled	80,000; 100,000	$20,000; $0
Workers Don't Get Skilled	$60,000; -$100,000	$40,000; $0

Race

- Racial discrimination in this model is a belief that White workers are more likely to be skilled than Black workers.

Employers believe:

- $P(S=1|W)=0.9$
- $P(S=1|B)=0.2$

- Consequence: Firms hire White workers who pass test, don't hire Black workers who pass test.
- Black workers have no incentive to invest in skills
- Firms beliefs are confirmed.

Escaping Discriminatory Equilibrium

- Can an individual worker change the equilibrium?
- Can an individual firm change the equilibrium?
- Would anything change if firms understood the game?

Escaping Discriminatory Equilibrium

- Equal Opportunity: Require firm to have equal standards for White and Black workers

- Affirmative Action: Require firm to hire equal percentage of White and Black applicants

Enriching the Model

- Multiple scores:
  - Make test passage non-binary: Range of scores from best to worst

- Multiple abilities:
  - Some workers have higher cost of study than others