Class 23: **Tuesday November 21**

TITLE Twentieth Century Mathematicians: Godel, Russell, Noether, Turing, Hilbert, etc
READING: Katz, pp. 874-903; Boyer & Mertzbach, pp. 548-586

SUMMARY
We will discuss other mathematical happenings in the Twentieth century!

NEXT: Twenty-First Century Mathematicians (Mirzakhani, Uhlenbeck, Simons, Wiles, Tao)
NEXT READING: Boyer, 587-599

Mathematicians and Mathematics of the Twentieth Century
The main themes of twentieth century historical developments in mathematics are: the effect and influence of emigration of mathematicians as a result of World War II, the splintering of mathematical philosophies, the development of statistics, the increase in the role of computers and computing in mathematics and society, acceptance of set theory and the popularization of topology.

COMPUTING
Alan Turing (1912-1954)
Turing was a British mathematician who is most well-known for his creation of the Turing machine, which answered one of Hilbert's famous problems. During World War II he was able to lead a team that cracked the German's "ENIGMA" code, which was a significant contribution to the Allied war effort. Computer Science's most prestigious award is named after him. Turing died by suicide eating a poisoned apple after years of government-decreed psychoanalysis and hormone treatments to cure his homosexuality, which was illegal in England at the time.

Donald Knuth (b. 1938)
Knuth is most well-known for his multi-volume work *The Art of Computer Programming* (TAOCP) and the creator of the TeX typesetting program. The first volume of TAOCP was published in 1968; it is currently in 4 volumes with another 4 planned. He won a scholarship to Case Western Reserve University in 1956. Professors were so impressed with his work that when he finished his undergraduate degree they bestowed a masters degree in addition. Knuth went on to get his PhD at Caltech and was a longtime professor at Stanford University.

John von Neumann (1903-1957)
One of the most prominent European emigres to the United States due to World War II was von Neumann, who was the son of a Jewish banker in Hungary. He was one of the founding members of the Institute for Advanced Study at Princeton University. He is most well-known for his work which led to the development of the modern computer.

MATHEMATICAL PHILOSOPHY, SET THEORY AND LOGIC

Bertrand Russell (1872-1970)
Russell was born in the United Kingdom and became one of the most prominent philosophers in the world. He is the leader of the "logicism" school of mathematical philosophy. He is the co-author (together with his former Cambridge PhD advisor **A.N. Whitehead**) of *Principia Mathematica*, a work which was intended to be as foundational to mathematical logic as Newton's *Philosophie Naturalis Principia Mathematica* was to mathematical physics. In logic he is well-known for his discovery of "Russell's Paradox" which demonstrated that the set theory created by **Georg Cantor** had logical gaps. Russell's paradox involves considering the set of all sets that are not members of themselves. This set appears to be a member of itself if and only if it is NOT a member of itself (but this is a contradiction since it violates the definition of the set).

Kurt Gödel (1906-1978)
Gödel was born in Austria and is considered to be one of the most significant logicians in history. Less than one year after receiving his PhD from the University of Vienna he published **two** "incompleteness theorems" which demonstrate the limitations of every formal axiomatic system that is capable of modeling basic arithmetic. The first states that "Any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F." The second theorem basically says that no consistent axiomatic system can be used to prove its own consistency.

David Hilbert (1862-1943)
Hilbert was one of the most famous mathematicians of the late 19^{th} and early 20^{th} centuries. He is considered the founder of the "formalism" philosophy of mathematics. The formalist philosophy is based on two ideas "all of mathematics can be generated from chosen finite system of axioms" and that "such an axiom system is provably consistent." He is also most well-known for his statement of 10 unsolved problems at the International Congress of Mathematicians in 1900 (which was expanded to 23 when published by the American Mathematical Society in 1902) that defined the contours of research mathematics for most of the 20^{th} century. The other schools of mathematical philosophy are the "intuitionist" and "logicist."

Paul Cohen (1934-2007)
Cohen was an American mathematician and Field's medalist who built on Godel's work to prove that the continuum hypothesis and the axiom of choice are independent from Zermelo-Fraenkel set theory (which is an axiomatic system that was developed to avoid Russell's paradox and other paradoxes). ZFC (Zermelo-Fraenkel set theory combined with the axiom of choice is one of the foundations of moderns set theory.)

STATISTICS
David Blackwell (1919-2010)
Blackwell was an African American mathematician who made significant contributions to many areas of applied mathematics (like information theory, probability theory) but especially Bayesian statistics. He was the seventh(!) African American to receive a PhD in Mathematics (at age 22), the first black tenured professor at UC Berkeley and the first black person inducted into the National Academy of Sciences.

EUROPEANS AND EUROPEAN EMIGRES

Amelie Emmy Noether (1882-1935)

Multiple contemporaries have described Emmy Noether as "the most important woman in the history of mathematics." Noether was born in Bavarian, Germany; her father was a mathematician and lecturer at the University of Erlangen. At the time, women were only allowed to audit university classes with instructor permission, but Noether was able to obtain and defend her Ph.D. dissertation on algebraic invariants in 1907. Noether made significant contributions to abstract algebra, mathematical physics and group theory. Hitler's rise in the early 1930s led Noether to emigrate to the United States where she was a professor at Bryn Mawr College until her death.

Paul Erdös (1913-1996)

Erdos is one of the most famous mathematicians of the twentieth century, primarily due to his eccentric, peripatetic lifestyle. For many, many years Erdös had no home, because he spent his time constantly travelling, for weeks or months at a time visiting other mathematicians and working with them to produce mathematical papers. He is so prolific (estimated to have been a co-author on at least 1500 papers!) that most mathematicians know their **Erdös number**, which is the "collaborative distance" between someone who has co-authored a paper with Erdös (which is a value of 1) to someone who has published a paper with someone who published a paper with Erdös (a value of 2).

Hermann Weyl (1885-1955)

Weyl was a member of the intuitionist school of mathematics (other members whose philosophies of mathematics would have included them include Brouwer, Kronecker and Poincare). The intuitionists believe that all of mathematics is to be built from finitely constructed operations on the natural numbers, which are derived from the human intuition of before and after and so on. Weyl was born in Germany but moved to the United Sates and was a member of the IAS soon after its founding and left Germany in the early 1930s (his wife was Jewish). He is considered one of the last universalists in mathematics, following the example of Poincare.

GROUPWORK

Let's have a discussion about the criteria for inclusion in a list of mathematicians of the twentieth (or twenty-first) century. What factors should be involved? What makes this task easier or harder than thinking about doing the same task for the fifteenth or sixteenth century?

Hilbert’s 23 Problems

At the Paris meeting of the International Congress of Mathematicians in 1900 Hilbert presented a list of 10 unsolved problems (in bold) and later **Mary Frances Winston Newson** (1867-1959, who became the first American woman to receive a PhD in mathematics at a European university in 1897 from the University of Göttingen, published a list of all 23 Hilbert problems in English in the *Bulletin of the American Mathematical Society* In 1902.

1. Cantor's problem of the cardinal number of the continuum.

2. The compatibility of the arithmetical axioms.

3. The equality of the volumes of two tetrahedra of equal bases and equal altitudes.

4. Problem of the straight line as the shortest distance between two points.

5. Lie's concept of a continuous group of transformations without the assumption of the differentiability of the functions defining the group.

6. Mathematical treatment of the axioms of physics.

7. Irrationality and transcendence of certain numbers.

8. *Problems of prime numbers (The "Riemann Hypothesis")*.

9. Proof of the most general law of reciprocity in any number field.

10. Determination of the solvability of a Diophantine equation.

11. Quadratic forms with any algebraic numerical coefficients

12. Extensions of Kronecker's theorem on Abelian fields to any algebraic realm of rationality

13. *Impossibility of the solution of the general equation of 7th degree by means of functions of only two arguments.*

14. Proof of the finiteness of certain complete systems of functions.

15. Rigorous foundation of Schubert's enumerative calculus.

16. *Problem of the topology of algebraic curves and surfaces.*

17. Expression of definite forms by squares.

18. Building up of space from congruent polyhedra.

19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?

20. The general problem of boundary values (Boundary value problems in PDE's).

21. Proof of the existence of linear differential equations having a prescribed monodromy group.

22. Uniformization of analytic relations by means of automorphic functions.

23. Further development of the methods of the calculus of variations.

The problems that are in *italics* are still unresolved (8^{th}, 13^{th} and 16^{th}); the others have been solved (either completely or partially) or are now thought to be too vaguely posed to be resolved (4^{th} and 23^{rd}).