Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 | Fowler 310 TR 1:30pm - 2:55pm

 | http://sites.oxy.edu/ron/math/395/23/

Class 6: Thursday September 14

TITLE Euclid's many contributions to mathematics
THIS READING: Katz, pp. 50-88; Boyer, pp. 90-108; Eves, pp. 140-155
NEXT READING: Katz, pp. 94-112; Barrow-Green, pp. 130-158, Eves, pp. 165-175

SUMMARY
The other books of the *Elements* cover an astonishing array of different fields of mathematics. We'll look at just a few of these amazing results today.

Book IV: Number Theory
The **Euclidean algorithm** for finding the greatest common divisor of two numbers.

Given two numbers a and b with $a>b$ subtract b from as many times as "possible."
If there is a remainder c (which must be less than b) then subtract c from b as many times as possible and continue until one ends with a number m which is the greatest common divisor. If this number is 1, then the two numbers a and b are said to be **relatively prime**.

The language that Euclid used was to say that m is the greatest common measure of a and b.

EXAMPLE
Use the Euclidean algorithm to find the greatest common divisor of 18 and 80.

One of the classic results appearing in Euclid's number theory books is the result that there are an infinite number of prime numbers. Additionally, the **fundamental theorem of arithmetic** was well-known to Euclid: ***Every number greater than 1 can be written as a <u>unique</u> product (up to the order of factoring) of prime numbers*****.**

Proposition VII-31. *Any composite number is measured by some prime number.*

Proposition VII-32. *Any number is either prime or is measured by some prime number.*

Proposition IX-20. *Prime numbers are more than any assigned multitude of prime numbers.*

PROOF

Let's look at Euclid's proof reproduced on page 80 of Katz. How would you improve it?

Book V: Ratio and Proportion
Theaetetus (417-369 BCE) and **Eudoxus** (408-355 BCE) were both Greek mathematicians that formalized the formulation of ratio and proportion. Eudoxus is well-known for his "Method of Exhaustion" and Theaetetus for his work on the regular polyhedra.

Euclid built on the work of these two and expanded it in Book V and a later book in the *Elements*.

Definition 5. *Magnitudes are said to be in the same ratio (or* ***proportional****), the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.*

In modern notation, we would say the magnitudes a, b, c and d are *in the same ratio* $a:b = c:d$ if for all positive integers m and n, $ma > nc$ then $mb > nd$; and similarly for **<** and **=**. This really means that when $ma>nc$ then $mb>nd$ and similarly for whenever $ma=nc$ this means that $mb=nd$ and whenever $ma<nc$ then $mb<nd$. In other words, the ratios a/c and b/d are always BOTH either <,> or = to the ratio n/m.

Definition

When $a:b = b:c$ this is said to mean that $a:c$ is the **duplicate ratio** of $a:b$.
In modern notation, we would say that the ratio of a to c is the square of the ratio a to b.

Continued proportion were sequences such that $a_1 : a_2 = a_2 : a_3 = a_3 : a_4 = ...$

EXAMPLE

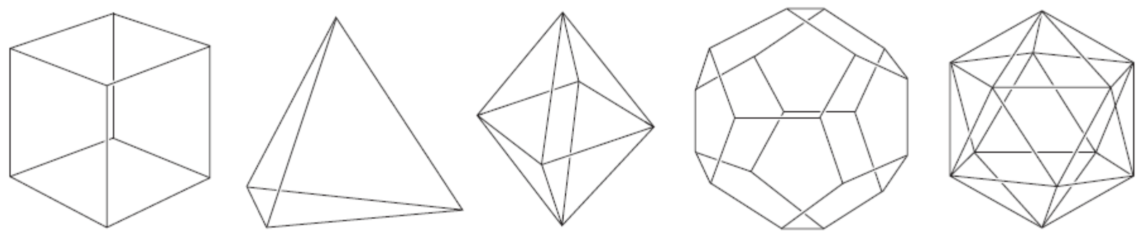
Proposition IX-35. *If as many numbers as we please are in continued proportion, and there is subtracted from the second and the last numbers equal to the first, then, as the excess of the second is to the first, so will the excess of the last be to all those before it*

Mathematically this is $(ar^n - a) : S_n = (ar - a) : a$

The Regular Polyhedra

Also known as the Platonic Solids, these are objects where each face is a regular polygon.

In Book XII, Euclid proved that these were the only regular polyhedra and showed how they could be constructed and inscribed inside a sphere.



Cube Tetrahedron Octahedron Dodecahedron Icosahedron

The Greeks were fascinated by the Platonic Solids and Plato assigned a natural Element to the four known at the time (4th century BCE):

cube=Earth, tetrahedron=Fire, octahedron=Air, icosahedron=Water.

Let's look at some other important characteristics of the Platonic solids.

	# of Faces	# of Vertices	# of Edges	# of Faces that meet at each Vertex	# of Sides of each Face
Tetrahedron					
Cube					
Octahedron					
Dodecahedron					
Icosahedron					

What patterns do you notice?

Which pairs of solids have the same number of edges? (These are known as the Dual Platonic Solids)

What's the relationship between the number of faces and the number of vertices for each of these pairs of solids?

What's the relationship between the number of faces that meet at each vertex and the number of sides of each face for these pairs of solids?