Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 — Fowler 310 TR 1:30pm - 2:55pm

 — http://sites.oxy.edu/ron/math/395/23/

Class 5: Tuesday September 12

TITLE Introduction to Euclid and the *Elements*
THIS READING: Katz, pp. 50-88; Boyer, pp. 90-108; Eves, pp. 140-155
NEXT READING: Katz, pp. 50-88; Boyer, pp. 90-108; Eves, pp. 140-155

SUMMARY
We will be introduced to Euclid (fl. 300 BCE), and the most famous mathematical book of all time: Euclid's *Elements*.

Online Version of Euclid's *Elements*
http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

Fame and Impact of *Elements*
Wikipedia describes it as:

> **Euclid's *Elements* has been referred to as the most successful and influential textbook ever written. It was one of the very earliest mathematical works to be printed after the invention of the printing press and has been estimated to be second only to the Bible in the number of editions published since the first printing in 1482, the number reaching well over one thousand. For centuries, when the quadrivium was included in the curriculum of all university students, knowledge of at least part of Euclid's Elements was required of all students. Not until the 20th century, by which time its content was universally taught through other school textbooks, did it cease to be considered something all educated people had read.**

Epistemology
How do we know what we know about Euclid and the *Elements*?

What do we know about Euclid?

What do we know about the *Elements*?

Postulates

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than to right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

EXAMPLE

Thoughts and reactions to the postulates?

Axioms

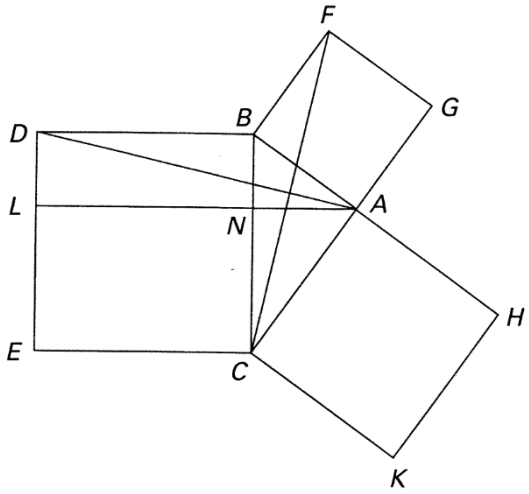
1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater that the part.

Exercise

Thoughts and reactions to the axioms?

Book I (of XIII) of the *Elements* is really all about getting to the point where Pythagoras' Theorem can be proved.

PROPOSITION I-47
In right-angled triangles the square on the hypotenuse is equal to the sum of the squares on the legs.



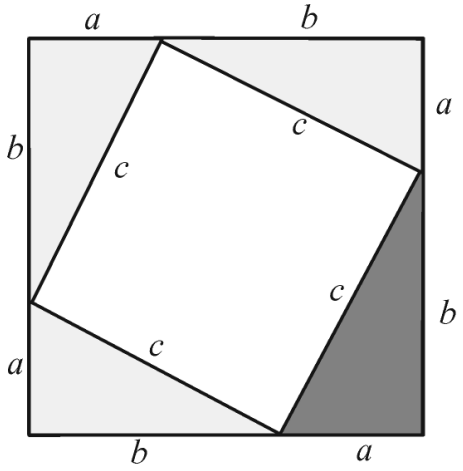
PROPOSITION I-46
On a given straight line to describe a square.
PROPOSITION I-4
If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal sides also equal, then the two triangles are congruent.
PROPOSITION I-41
If a parallelogram has the same base with a triangle and is in the same parallels, the parallelogram is double the triangle.
PROPOSITION I-27
If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.
same parallels, the parallelogram is double the triangle.
PROPOSITION I-16
In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.
PROPOSITION I-34
In parallelograms the opposite sides and angles equal one another, and the diameter bisects the areas.
PROPOSITION I-29
A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

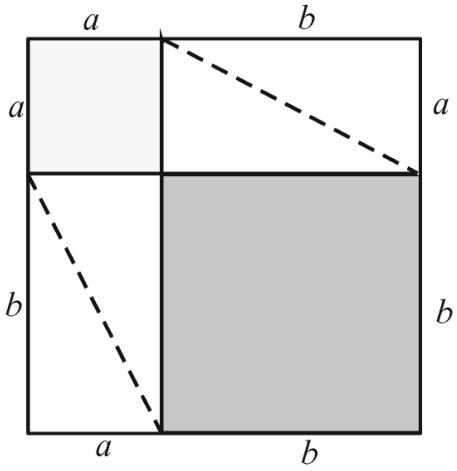
GroupWork
Let's repeat Katz's discussion (pp. 54-59) and work backwards through the propositions (theorems) that one needs in order to prove Pythagoras' Theorem.

Alternate Proof(s) of Pythagoras Theorem

G. Donald Allen claims there are over 300 published proofs of this theorem, and gives the following very pretty one

$$\begin{aligned}(a+b)^2 &= c^2 + 4(\frac{1}{2}\,ab)\\ a^2 + 2ab + b^2 &= c^2 + 2ab\\ a^2 + b^2 &= c^2\end{aligned}$$





This proof is based upon Books I and II of Euclid's *Elements*, and is supposed to come from the figure to the right. Euclid allows the decomposition of the square into the two boxes and two rectangles. The rectangles are cut into the four triangles shown in the figure.

Then the triangle are reassembled into the first figure.

Exercise

Explain how the images work as a proof of Pythagoras Theorem.