Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 — Fowler 310 TR 1:30pm - 2:55pm

 `http://sites.oxy.edu/ron/math/395/23/`

Homework #9

ASSIGNED: Tue Oct 31 2023 — **DUE: Tue Nov 7 2023**

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (Adapted from **Class Worksheet #15**.) Show that the solution to the Euler equation $x^2y'' + 6xy' + 6y = 0$ with boundary conditions $y(1) = 1$ and $y(-1) = 3$ is $y = 2/x^2 - 1/x^3$.

2. (Adapted from **Class Worksheet #16**.) Recall the result of Descartes that the sum of the reciprocal of the roots of the n^{th}-degree polynomial $1 + c_1x + c_2x^2 + c_3x^3 + \ldots + c_nx^n$ is $-c_1$. Prove the result for the $n = 2$ quadratic case.

3. (Adapted from **Class Worksheet #16**.) The sum of the reciprocal of the squares of all the **odd** positive integers is known. Show that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$. [HINT: Replicate Euler's solution of the Basel problem but use a series expansion for $\cos(x)$]

4. (Adapted from **Katz, Chapter 18, p. 637, #17**.) Find the natural logarithm of the **three** cube roots of 1. [HINT: You will need to use the multi-valued complex logarithm $\log(z)$.]

5. (Adapted from **Katz, Chapter 18, p. 638, #27**.) Show that the cone with minimal surface area S given a fixed volume V is one where $h = r\sqrt{2}$ and thus $S = \sqrt{3}\pi r^2$. [HINT: recall a cone with radius r and height h has $V = \frac{1}{3}\pi r^2 h$ and $S = \pi r\sqrt{h^2 + r^2}$.]

6. (Adapted from **Eves, Chapter 10, p. 463, #12.11)**. *This problem can also be done as a Formal Homework.* Consider the famous curve "the witch of Agnesi" (in Figure 1) given by the equation $y = \dfrac{a^3}{x^2 + a^2}$ where a is a given parameter. The curve can be described parametrically by $x = at, y = \dfrac{a}{1+t^2}$ for $-\infty \leq t < \infty$.

   (a) Show algebraically that the curve is symmetric about the y-axis and has the x-axis as an asymptote.

   (b) Show that the area between the curve and the x-axis is exactly 4-times the area of the circle it encloses.

   (c) Show that the volume of the shape formed by rotating the curve around its asymptote is exactly equal to $\pi^2a^3/2$

   (d) Show that the angle between the asymptote and a line from the origin to either inflexion point on the curve is $\arctan(3\sqrt{3}/4)$.

   (e) Generate a figure illustrating the result in part (d).

(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in LaTeX to generate mathematical symbols, uses complete sentences and is in narrative form; the work is done individually.

1. (Adapted from **Eves, Chapter 10, p. 463, #12.11).** Consider the famous curve "the witch of Agnesi" (in Figure 1) given by the equation $y = \dfrac{a^3}{x^2 + a^2}$ where a is a given parameter. The curve can be described parametrically by $x = at, y = \dfrac{a}{1 + t^2}$ for $-\infty \leq t < \infty$.

   (a) Show algebraically that the curve is symmetric about the y-axis and has the x-axis as an asymptote.

   (b) Show that the area between the curve and the x-axis is exactly 4-times the area of the circle it encloses.

   (c) Show that the volume of the shape formed by rotating the curve around its asymptote is exactly equal to $\pi^2 a^3/2$

   (d) Show that the angle between the asymptote and a line from the origin to either inflexion point on the curve is $\arctan(3\sqrt{3}/4)$.

   (e) Generate a figure illustrating the result in part (d).

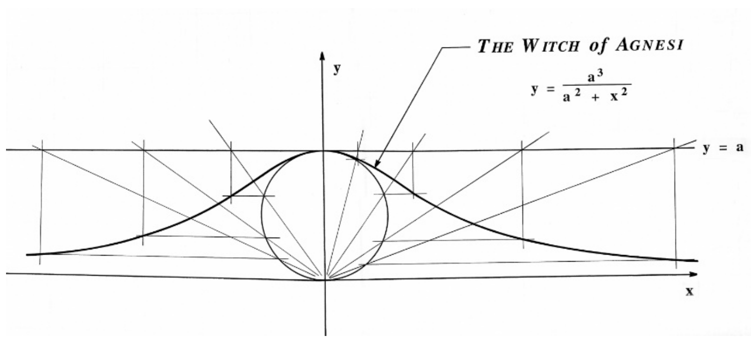

Figure 1: The "witch" of Agnesi