Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 — Fowler 310 TR 1:30pm - 2:55pm

 — http://sites.oxy.edu/ron/math/395/23/

Homework #8

ASSIGNED: Tue Oct 24 2023 — **DUE: Tue Oct 31 2023**

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (Adapted from **Katz, Chapter 16, p. 580, #13**.) **(a)** Use modern Calculus techniques to integrate $y = \dfrac{ax^{n-1}}{e + fx^n}$. **(b)** Show that your answer is identical to Newton's, which is $z = \frac{1}{n}S$ where S is the area under the hyperbola $v = \dfrac{a}{e + fu}$.

2. (Adapted from **Katz, Chapter 16, p. 580, #16**.) Use Newton's method of fluxions to show that the fluxion of $y = \dfrac{1}{x}$ is $\dfrac{\dot{y}}{\dot{x}} = \dfrac{-1}{x^2}$.

3. (Adapted from **Katz, Chapter 16, p. 580, #28**.) Use Leibniz's method of differentials to show that when $y = x^x$, the relationship between differentials is $dy = x^x(\log(x) + 1)dx$.

4. (Adapted from **Katz, Chapter 16, p. 580, #30**.) Derive the power series for the natural logarithm function by solving the initial value problem $\dfrac{dy}{dx} = \dfrac{1}{1+x}$ with $y(0) = 0$, assuming that $y = \displaystyle\sum_{k=0}^{\infty} a_k x^k$ and use the method of undetermined coefficients to obtain $a_0, a_1, a_2, a_3, a_4, \ldots$

5. (Adapted from **Class Worksheet #13**). *This problem can also be done as a Formal Homework.* Recall Leibniz found the result that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \dfrac{1}{11} + \ldots$.

   (a) Show that π can be written as the infinite series $\mathcal{S} = 4\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$

   (b) Use an appropriate test to show that the series $\mathcal{S}$ converges.

   (c) Considering that $\mathcal{S} = \displaystyle\lim_{n\to\infty} S_n$ where S_n are the partial sums of the first n terms of $\mathcal{S}$, use a computer (Desmos! Excel! Python!) to evaluate S_{10}, S_{20}, S_{40}, S_{80}, S_{100} and S_{1000} (or more!). Tabulate your results.

   (d) Plot your results on a graph to obtain a relationship between n and the difference between S_n and π, i.e. between n and $\mathcal{E}_n$ where $\mathcal{E}_n = |S_n - \pi|$.

   (e) What's the value of $\displaystyle\lim_{n\to\infty} \mathcal{E}_n$? How would you describe this result using "big Oh" notation, $\mathcal{O}$?

   (f) Comment on the observation that I made in class that "This Leibniz formula for estimating π is very slow." Do you agree or disagree?

(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in LaTeX to generate mathematical symbols, uses complete sentences and is in narrative form; the work is done individually.

1. (Adapted from **Class Worksheet #13).** *This problem can also be done as a Formal Homework.* Recall Leibniz found the result that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$.

   (a) Show that π can be written as the infinite series $\mathcal{S} = 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$

   (b) Use an appropriate test to show that the series $\mathcal{S}$ converges.

   (c) Considering that $\mathcal{S} = \lim_{n\to\infty} S_n$ where S_n are the partial sums of the first n terms of $\mathcal{S}$, use a computer (Desmos! Excell! Python!) to evaluate S_{10}, S_{20}, S_{40}, S_{80}, S_{100} and S_{1000} (or more!). Tabulate your results.

   (d) Plot your results on a graph to obtain a relationship between n and the difference between S_n and π, i.e. between n and $\mathcal{E}_n$ where $\mathcal{E}_n = |S_n - \pi|$.

   (e) What's the value of $\lim_{n\to\infty} \mathcal{E}_n$? How would you describe this result using "big Oh" notation, $\mathcal{O}$?

   (f) Comment on the observation that I made in class that "This Leibniz formula for estimating π is very slow." Do you agree or disagree?