Homework #6

ASSIGNED: Thu Oct 5 2023   **DUE: Fri Oct 13 2023**

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (Adapted from **Katz, Chapter 9, p. 319, #12**) Use Alhazen's formula from Worksheet #10 to derive the formula

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \frac{n^2(n+1)^2}{4}$$

2. (**Katz, Chapter 9, p. 319, #14**) Show, using the formulas for sums of fourth powers and squares, that

$$\sum_{i=1}^{n-1} (n^4 - 2n^2 i^2 + i^4) = \frac{8}{15}(n-1)n^4 + \frac{1}{30}n^4 - \frac{1}{30}n = \frac{8}{15}n^5 - \frac{1}{2}n^4 - \frac{1}{30}n$$

3. (**Katz, Chapter 7, p. 227, #14**) Show that the diameter D of the largest circle that can be inscribed in a right triangle with legs a and b and hypotenuse c is given by $D = \dfrac{2ab}{a+b+c}$. (This is a generalization of problem 16 of chapter 9 of the *Nine Chapters*, which uses the specific 8-15-17 triangle.)

4. (**Katz, Chapter 8, p. 279, #12**) Another problem from Mahavira: Of a collection of mango fruits, the king took 1/6; the queen took 1/5 of the remainder, and three chief princes took 1/4, 1/3, 1/2 of what remained at each step; and the youngest child took the remaining three mangoes. O you, who are clever in working miscellaneous problems on fractions, give out the measure of that collection of mangoes.

5. **African Strip Patterns** Determine which of the seven possible symmetries each of the four African frieze patterns has.

A.   C.

C.   D.
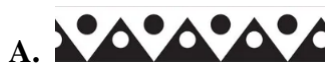
(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in LaTeX to generate mathematical symbols, uses complete sentences and is in narrative form; the work is done individually.

1. (Adapted from **Katz, Chapter 9, p. 319, #21**)) Sharaf al-Dīn al-Tūsī analyzed the solutions of various cubic equations (See Katz Figure 9.13 on page 291). Let's consider $x^3 + d = cx$ where d and c are given parameters.

   (a) Show using calculus that $\left(\sqrt{\frac{c}{3}}, \sqrt{\frac{4c^3}{27}} \right)$ is the location of the maximum of the function $f(x) = x(c - x^2)$.

   (b) (Use calculus to) Sketch the graph of $y = x^3 - cx + d$

   (c) Determine the conditions on the parameters c and d for which there are 0, 1 and 2 roots

   (d) Using your answers above, how many solutions does the equation $x^3 + 1 = 2x$ have?

   (e) Using your answers above, how many solutions does the equation $x^3 + 2 = x$ have?

   (f) Depict your answers to the last two problems graphically. [HINT: Use Desmos!]