Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 | Fowler 310 TR 1:30pm - 2:55pm

 `http://sites.oxy.edu/ron/math/395/23/`

Homework #5

ASSIGNED: Thu Sep 28 2023 **DUE: Fri Oct 6 2023**

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (**Katz, Chapter 10, p. 359, #3**) A hare is 150 paces ahead of a hound that is pursuing him. If the hound covers 10 paces each time the hare covers 6, in how many paces will the hound overtake the hare?

2. (Adapted from **Katz, Chapter 10, p. 359, #36**) Jordanus de Nemore knew the following result: "If the sum of two numbers is given together with the product of their squares, then each of them is determined."

   (a) Solve Jordanus's problem when $x + y = 9$ and $x^2y^2 = 324$.

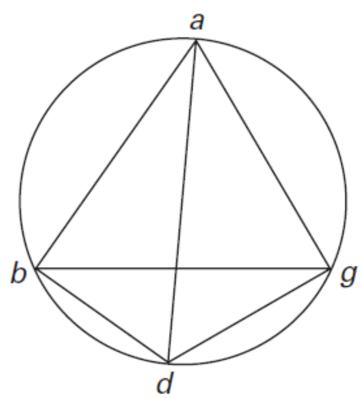
   (b) Consider the general case where $x + y = a$ and $x^2y^2 = b^2$ and show that the general solution to the problem is $x = \frac{1}{2}(a + \sqrt{a^2 - 4b})$ and $y = \frac{1}{2}(a^2 - \sqrt{a - 4b})$. [HINT: $(x - y)^2 = (x + y)^2 - 4xy$]

3. (Adapted from **Katz, Chapter 10, p. 359, #31**)) Consider the Fibonacci series F_n given by $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$

   (a) Show that $F_{n+1}F_{n-1} = F_n^2 - (-1)^n$

   (b) Show that $\lim_{n\to\infty} \dfrac{F_n}{F_{n-1}} = \dfrac{1+\sqrt{5}}{2}$

4. (Adapted from **Katz, Chapter 10, p. 359, #11**)) From Leonardo's *Practica geometriae*: Given the quadrilateral inscribed in a circle with $ab = ag = 10$ and $bg = 12$, find the diameter ad of the circle. [NOTE: The line ae is perpendicular to bg where e is the intersection of bg and ad]



(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in $\LaTeX$ to generate mathematical symbols, uses complete sentences and is in narrative form; the work is done individually.

1. (Adapted from **Katz, Chapter 10, p. 359, #8**)) The *Artis cuiuslibet consummatio* claimed that the formula $A_n = \frac{3n^2-n}{2}$ gave the area of a pentagon of side n.

   (a) Show, instead, that A_n provides a formula for the nth pentagonal number. [HINT: Show that the nth pentagonal number can be written as $1+4+7+10+\ldots+(3n-2)$]

   (b) Show that the exact area of a regular pentagon with edge length r is $P_r = 5r^2\frac{\tan(3\pi/10)}{4}$

   (c) Find the exact area P_r of regular pentagons with sides $r = 1, 2, 3,$

   (d) Compare your answers for P_r with the first $n + 1$-st pentagonal numbers given by A_n. How close an approximation does A_{n+1} provide for P_r?