Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 — Fowler 310 TR 1:30pm - 2:55pm

 — http://sites.oxy.edu/ron/math/395/23/

Homework #4

ASSIGNED: Thu Sep 21 2023 — **DUE: Fri Sep 29 2023**

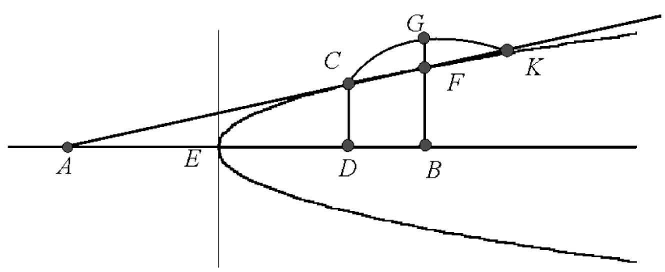
The homework consists of questions for which you can produce informal and (optional) formal responses.

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (Adapted from **Katz, Chapter 4, p.129, #19**) Use calculus to prove *Conics* **I-33**.

   Proposition I-33. If AC is constructed, where $|AE| = |ED|$, then AC is tangent to the parabola.



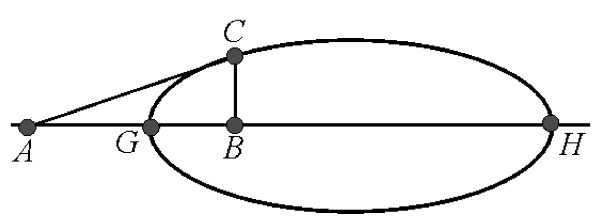
   [(HINT: In modern coordinates, you need to show that the tangent line to the parabola $y^2 = px$ at point C (x_0, y_0) has an x-intercept at $(-x_0, 0)$.]

2. (Adapted from **Katz, Chapter 4, p.129, #20**) Use calculus to prove *Conics* **I-34**.

   Proposition I-34. (ellipse) Choose A so that

$$\frac{|AH|}{|AG|} = \frac{|BH|}{|BG|}$$

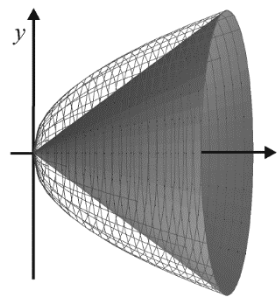
   Then AC is tangent to the ellipse at C.



   [(HINT: In modern coordinates, you need to show that the tangent line to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point C (x_0, y_0) has an x-intercept at $(x_0/a^2, 0)$.]

3. (Adapted from **Worksheet #7 (Tuesday September 19), p.4**) Use Calculus to prove Proposition 21 in *Of Conoids and Spheroids* by Archimedes:

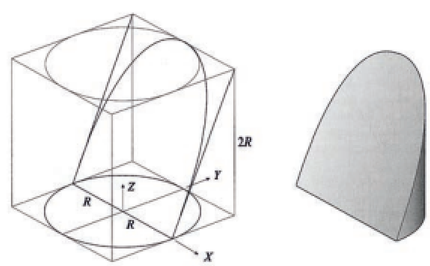
   > "Any segment of a paraboloid of revolution is half as large again as the cone or segment of a cone which has the same base and the same axes."



   HINT: Let V_{cone} be the volume of the cone of radius R and height h and V_{para} be the volume of the paraboloid. Show that $V_{para}/V_{cone} = 3/2$.

4. (Adapted from **Worksheet #7 (Tuesday September 19), p.4**) Use Multivariable Calculus to prove Proposition 14 in *The Method* by Archimedes:

   > "If a cylinder is inscribed in a rectangular parallelepiped with square base, and if a plane is drawn through the center of the circle at the base of the cylinder and through one side of the square forming the top of the parallelepiped, then the segment of the cylinder cut off by this plane has a volume equal to one sixth of the entire parallelepiped."



   HINT: Let the parallepiped in this case be a cube with side $2R$. Show that the volume V of the given shape (a.k.a "the hoof") is given by the integral $\int_{-R}^{R}\int_{0}^{\sqrt{R^2-x^2}}\int_{0}^{2y} dz\, dy\, dx.$ Use Fubini's Theorem to write down a different double/triple integral for V and show it also equals $V = 4R^3/3$

(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in LaTeX to generate mathematical symbols, uses complete sentences and is in narrative form; the work is done individually.

1. The "trinity of ancient Greek mathematicians" is said to be Euclid, Archimedes and Apollonius. Pick one of these mathematicians (or someone else!) and write a 2-3 pages (400-600 words) essay that argues why your selection is the greatest of these ancient Greek mathematicians (clearly state the criteria you used for your choice). Make sure to also describe mathematically (i.e. include equations and/or figures) at least one of your choice's key mathematical contributions and their overall historical significance.