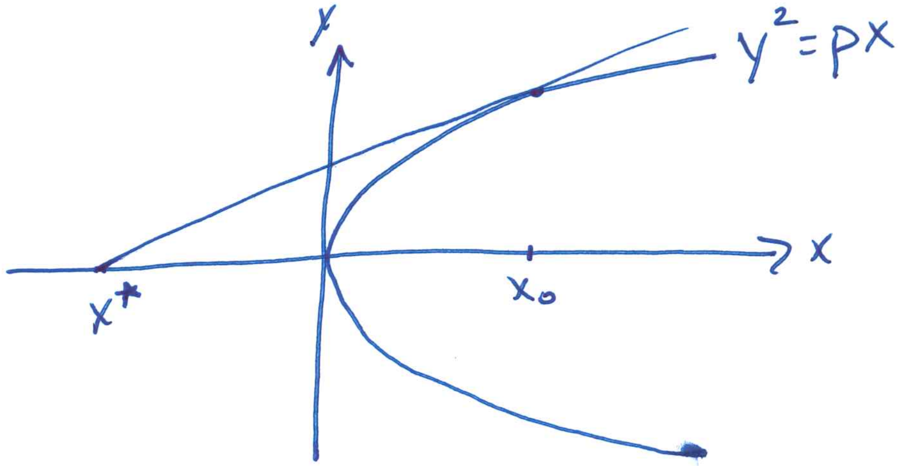
HW #4

1.



Find eq^n of tangent line at (x_0, y_0)

$$2y\frac{dy}{dx} = p \Rightarrow \frac{dy}{dx} = \frac{p}{2y_0} \quad \text{at } (x_0, y_0)$$

$$y - y_0 = \frac{p}{2y_0}(x - x_0)$$

when $x = x^*$, $y = 0$ so x^* is the x-intercept

$$0 - y_0 = \frac{p}{2y_0}(x^* - x_0)$$

$$-2y_0^2 = px^* - px_0$$

$$-2y_0^2 + px_0 = px^*$$

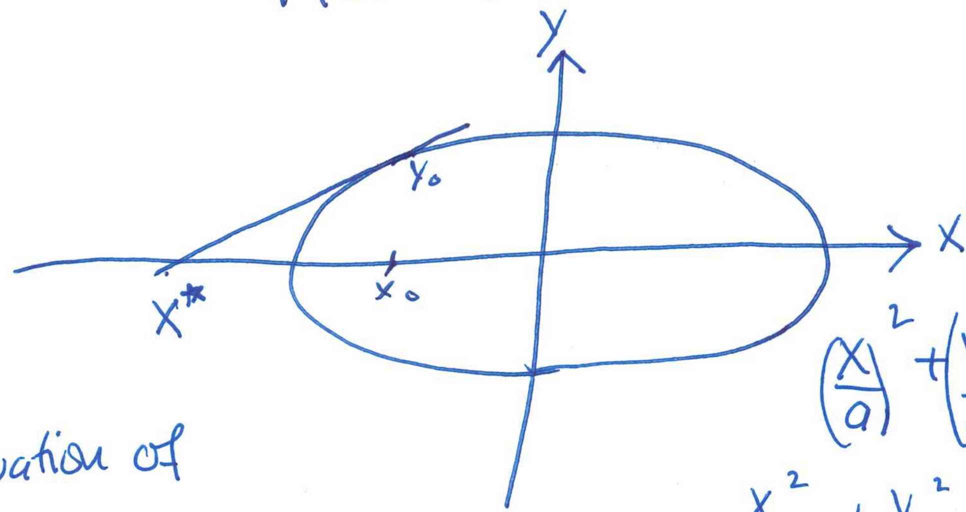
$$\frac{-2y_0^2}{p} + x_0 = x^* \Rightarrow x^* = -2x_0 + x_0 = -x_0$$

But $y^2 = px$ so $y_0^2 = px_0$

so $\frac{-2y_0^2}{p} = -2x_0$

HW #4

2.



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$$

$$\Rightarrow b^2 x_0^2 + a^2 y_0^2 = a^2 b^2$$

Find equation of Tangent line at (x_0, y_0) of ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{a^2} \cdot \frac{b^2}{2y}$$

At (x_0, y_0)

$$\frac{dy}{dx} = -\frac{x_0 b^2}{y_0 a^2}$$

Eq^n of tangent line

$$y - y_0 = \left(-\frac{x_0 b^2}{y_0 a^2}\right)(x - x_0)$$

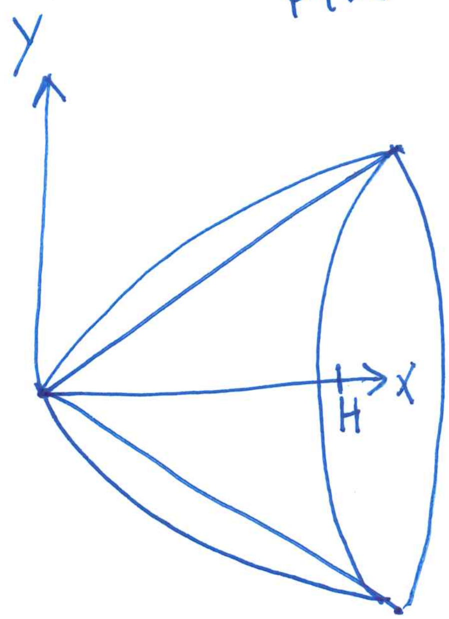
When $x = x^*$, $y = 0$

$$0 - y_0 = -\left(\frac{x_0 b^2}{y_0 a^2}\right)(x^* - x_0) \Rightarrow \frac{y_0^2 a^2}{x_0 b^2} = x^* - x_0$$

$$x^* = x_0 + \frac{y_0^2 a^2}{x_0 b^2} = \frac{x_0^2 b^2 + y_0^2 a^2}{x_0 b^2} = \frac{a^2 b^2}{x_0 b^2} = \frac{a^2}{x_0} \checkmark$$

#3

HW #4



$$y^2 = R_1 x$$

$$x = H,\ y^2 = \frac{R^2}{H} x$$

$$V = \pi \int_0^H \left(\frac{R}{H} x\right)^2 dx$$

$$V_{cone} = \pi \frac{R^2}{H^2} \frac{x^3}{3} \Big|_0^H = \frac{1}{3} R^2 H \pi$$

$$V_{par} = \pi \int_0^H \frac{R^2 x}{H}\, dx$$

$$= \pi \frac{R^2}{H} \frac{H^2}{2} = \frac{\pi R^2 H}{2}$$

$$\frac{V_{cone}}{V_{par}} = \frac{\frac{1}{3}\pi R^2 H}{\frac{1}{2}\pi R^2 H} = \frac{2}{3}$$

$$V_{par} = \frac{3}{2} V_{cone}$$

HW #4

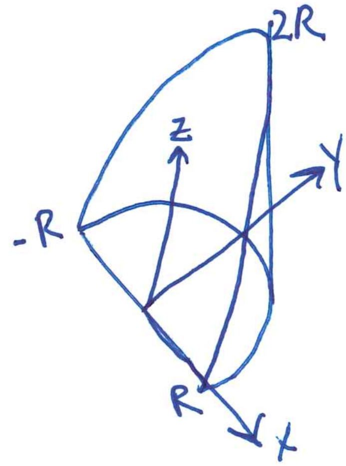
#4

$$V=\int_{-R}^{R}\int_{0}^{\sqrt{R^2-x^2}}\int_{0}^{2y} dz\,dy\,dx = \int_{-R}^{R}\int_{0}^{\sqrt{R^2-x^2}} z\Big|_0^{2y}\,dy\,dx$$

$$=\int_{-R}^{R}\int_{0}^{\sqrt{R^2-x^2}} 2y\,dy\,dx = \int_{-R}^{R} y^2\Big|_0^{\sqrt{R^2-x^2}} dx = \int_{-R}^{R}\left(\sqrt{R^2-x^2}\right)^2 dx$$

$$=\int_{-R}^{R} R^2-x^2\,dx = R^2x-\frac{x^3}{3}\Big|_{-R}^{R} = 2\left(R^2x-\frac{x^3}{3}\right)\Big|_0^{R}$$

$$=2\left(R^3-\frac{R^3}{3}\right)=\left(\frac{2R^3}{3}\right)2=\frac{4R^3}{3}$$



Change order to dz dx dy

$$V=\int_{0}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\int_{0}^{2y} dz\,dx\,dy$$

$$=\int_{0}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} 2y\,dx\,dy = \int_{0}^{R} 2yx\Big|_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} dx = \int_{0}^{R} 2y\cdot 2\sqrt{R^2-y^2}\,dy$$

$$=2\int_{0}^{R} 2y\sqrt{R^2-y^2}\,dy = 2\int_{R^2}^{0} -\sqrt{u}\,du = 2\int_{0}^{R^2}\sqrt{u}\,du = \frac{4}{3}u^{3/2}\Big|_0^{R^2}$$

$$=\frac{4R^3}{3}$$

$u=R^2-y^2$ $y=0,\ u=R^2$

$du=-2y\,dy$ $y=R,\ u=0$