Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 | Fowler 310 TR 1:30pm - 2:55pm

 | `http://sites.oxy.edu/ron/math/395/23/`

Homework #3

ASSIGNED: Thu Sep 14 2023 **DUE: Thu Sep 21 2023**

The homework consists of questions for which you can produce informal and (optional) formal responses.

Informal Homework Responses

The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be (**neatly!**) handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained. Clearly indicate your answers.

1. (**Katz, Chapter 3, p.91, #19**) Use the Euclidean algorithm to (a) find the greatest common divisor of 963 and 657; (b) of 2689 and 4001.

2. (**Katz, Chapter 3, p.91, #26**) Use Eudoxus's definition to prove Proposition V-16: "If $a : b = c : d$, then $a : c = b : d$."

3. (**Katz, Chapter 3, p.91, #27**) Construct geometrically the solution of $8 : 4 = 6 : x$.

4. (**Katz, Chapter 3, p.91, #28**) Solve geometrically the equation $\frac{9}{x} = \frac{x}{5}$ by beginning with a semicircle of $9 + 5 = 14$.

5. (Adapted from **Eves, Chapter 3, p.97, #3.6**) Show that if $(a, a+1, c)$ is a Pythagorean triple, so is $(3a + 2c + 1, 3a + 2c + 2, 4a + 3c + 2)$

6. (Adapted from **Eves, Chapter 3, p.101, #3.13**) Show that if a cube (a solid having 6 congruent squares as faces) and an octahedron (a solid having 8 congruent faces) are inscribed in the same sphere, the cube has the larger volume than the octahedron.

7. (Adapted from **Eves, Chapter 3, p.102, #3.16**) The following result is known as Nichomachus' Theorem:

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$$

   (a) Write the expression of Nichomachus' Theorem using $\sum$ notation.

   (b) Prove (by induction) that $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$.

   (c) Provide a geometric "proof" (or interpretation) of Nichomachus' theorem.

(Optional) Formal Homework Responses

Choose the following problem as a formal homework response. This means that the solution is written up in LaTeX with complete sentences in narrative form and the work is done individually.

1. (Adapted from **Eves, Chapter 5, p.159, #5.9**) Consider the sequence of regular n-sided polygons with $n = 3, 4, 5, 6, \ldots, \infty$.

   (a) $n = 3$ Show that the area of the equilateral triangle is equal to half the product of its perimeter and the radius of its inscribed circle.

   (b) $n = 4$ Show that the area of the square is equal to half the product of its perimeter and the length of its apothem (the perpendicular distance from the center of the square to each side).

   (c) $n = 5$ or $n = 6$. Show that the area of the pentagon (i.e. if you pick $n = 5$) OR the hexagon (i.e. you pick $n = 6$) is equal to half the product of its perimeter and the length of its apothem (the perpendicular distance from the center of the object to each polygonal side).

   (d) $n \to \infty$ The infinite-sided polygon is a circle. What is the corresponding formula for the area of the "infinite-sided regular polygon" a.k.a. circle corresponding to what you did in parts (a), (b) and (c)?