Special Topics in Advanced Math: *History of Mathematics*

Math 395 Fall 2023 Fowler 310 TR 1:30pm - 2:55pm

 http://sites.oxy.edu/ron/math/395/23/

Homework #2

ASSIGNED: Thu Sep 7 2023 **DUE: Thu Sep 14 2023**

The homework consists of questions for which you can produce formal and/or informal responses.

Informal Homework Responses

1. The first set of questions counts as one informal homework response. These problems can be worked on in groups with each student submitting their own solutions. They can be handwritten and can provide the "usual" amount of work to demonstrate how the solutions were obtained.

   (a) (**Katz, Chapter 2, p.47, #2**) Represent 8/9 as a sum of distinct unit fractions. Express the result in the Greek notation. (Note that the answer to this problem is not unique.)

   (b) (**Katz, Chapter 2, p.47, #8**) Show that the n^{th} triangular number T_n is represented algebraically as $T_n = \frac{n(n+1)}{2}$ and therefore that an oblong number is double a triangular number.

   (c) (**Katz, Chapter 2, p.47, #9**) Show algebraically that any square number is the sum of two consecutive triangular numbers.)

   (d) (**Katz, Chapter 2, p.47, #11**) Show that in a Pythagorean triple, if one of the terms is odd, then two of them must be odd and one even.

   (e) (Adapted from **Katz, Chapter 2, p.48, #18**) Give one example of each of the four types of syllogisms mentioned in the text.

   (f) (Adapted from **Katz, Chapter 2, p.48, #20**) Use truth tables to show that the Hypothetical syllogism and the Alternative syllogism are tautologies.

   (g) (Adapted from **Eves, Chapter 3, p.94, #3.2**) Show that the numbers 1184 and 1210 are **amicable**, that is the sum of their proper divisors is the other.

   (h) (Adapted from **Eves, Chapter 3, p.94, #3.3**)

      (i) List the first four hexagonal numbers.

      (ii) Show both geometrically and algebraically that any oblong number is twice a triangular number.

      (iii) Denoting the oblong number $n(n+1)$ as O_n show geometrically and algebraically that $O_n - S_n = n$, the difference between the n^{th} oblong number and n^{th} square number is n.

(Optional) Formal Homework Responses

Choose ONE of the following problems as a formal homework response. This means that the solution is written up in LaTeX with complete sentences in narrative form and the work is done individually.

1. There are three famous ancient problems of antiquity which Greek mathematicians obsessed over for centuries: **Squaring the Circle**, **Doubling the Cube**, and **Trisecting the Angle**. Pick one of the problems and write a 2-3 pages (400-600 words) essay that describes the problem, discusses its history and significance and provides a mathematical explanation (doesn't need to be a formal proof) for why its solution is impossible (at least as posed by the Greeks).

2. (Adapted from **Eves, Chapter 3, p.94, #3.4**) The *Eudemian summary* says that in Pythagoras' time there were three means. the **arithmetic**, the **geometric**, and the **subcontrary**, which was changed to **harmonic** by Archytas and Hippasus. These means are defined as

$$A = \frac{a+b}{2}, \qquad G = \sqrt{ab}, \qquad H = \frac{2ab}{a+b}$$

   (a) Show that $A \geq G \geq H$, with equality holding when $a = b$

   (b) Show that $a : A = H : b$. This was known as the "musical proportion."

   (c) Show that $1/(H-a) + 1/(H-b) = 1/a + 1/b$

   (d) Show that if a,b,c are in harmonic progression, so are $\dfrac{a}{b+c}$, $\dfrac{b}{c+a}$, and $\dfrac{c}{a+b}$