Senior Colloquium: *History of Mathematics*

Math 400 Spring 2020 — Fowler 310 T 1:30pm - 2:55pm

 — http://sites.oxy.edu/ron/math/400/20/

QUIZ #1: April 7, 2020

NAME:_____

Directions: This quiz is intended to take (at most) 60 minutes. This is a closed book, closed notes quiz. There are 5 parts spread over 5 pages. Please look at all the questions before answering any of them. You should use complete English sentences as much as possible and CLEARLY indicate your final answers from your scratch work.

Pledge: "I pledge my honor as an Occidental College student and human being that I have followed the rules and directions stated above to the letter and in spirit."

□ Select Here (or Provide Signature Below) To Confirm Your Acceptance of the Pledge.

Question	Score	Maximum
Math: Part I		15
Math: Part II		10
History: Part I		10
History: Part II		5
History: Part III		10
TOTAL		**50**

MATH PART I: (15 points)

The goal of this problem is to show that $\mathcal{I} = \int_0^{2\pi} \frac{d\theta}{b + \cos\theta} = \frac{2\pi}{\sqrt{b^2 - 1}}, b > 1.$

(a) Use the substitution $z = e^{i\theta}$ to show that $\mathcal{I}$ can be re-written as $\mathcal{J} = \frac{1}{i} \oint_{|z|=1} \frac{2\, dz}{z^2 + 2bz + 1}$

(b) Use Cauchy's Residue Calculus to compute *J*. **Explain how computing *J* allows you to evaluate *I* and why the condition b>1 is important.**

MATH PART II: (10 points)
The goal of this problem is to demonstrate the properties of the Laguerre polynomials $L_n(x)$**, that satisfy the Laguerre differential equation:** $xy''+(1-x)y'+ny=0$.

(a) Use the Rodrigues' formula for the Laguerre polynomials, $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})$, to confirm that $L_0(x) = 1$, $L_1(x) = 1 - x$, and to find $L_2(x)$.

(b) To confirm your expression for $L_2(x)$ found in part (a) is correct use the recurrence relation $(n+1)L_{n+1}(x)= (2n+1-x)L_n(x)-nL_{n-1}(x)$.

(c) Verify that your expressions for $L_0(x)$, $L_1(x)$, and $L_2(x)$ satisfy the Laguerre differential equation $xy''+(1-x)y'+ny=0$.

HISTORY PART I: LONG-ANSWER QUESTION (10 points).
The goal of this problem is to provide you an opportunity to demonstrate your knowledge of important mathematical figures of the 18^{th} and 19^{th} centuries.
If Euler or Gauss had never been born, whom do you think would be considered the most important (i.e. historically significant) mathematician of the 18^{th} and 19^{th} centuries? WRITE LEGIBLY and provide 1-3 paragraphs (i.e. each with multiple sentences) to support your answer! Be sure to include biographical and mathematical information about the person you select.

HISTORY PART II: MATCH QUESTION (5 points)
Match the concept, symbol or equation with the name of the one Mathematician most closely associated with it. (You can write down the number next to the letter.)

A: $P_0(x)=1, P_1(x)=x, P_2(x)=\frac{1}{2}(3x^2-1), \ldots$

B: The continuum hypothesis

C: $e^{\pi i} + 1 = 0$

D: The fundamental theorem of algebra

E: A function which is nowhere continuous

1. **Leonhard Euler**
2. **Carl Friedrich Gauss**
3. **Joseph-Louis Lagrange**
4. **Pierre-Simon Laplace**
5. **Adrien-Marie Legendre**
6. **Augustin-Louis Cauchy**
7. **Joseph Fourier**
8. **Johann Dirichlet**
9. **Evariste Galois**
10. **Georg Cantor**

A_____ B_____ C_____ D_____ E_____

HISTORY PART III: SHORT-ANSWER QUESTIONS (10 points)
Write down whether the following sentences are either TRUE or FALSE (by writing a T or F on the blank line) for 1 point. USE THE SPACE BETWEEN POINTS TO EXPLAIN YOUR ANSWER FOR 1 MORE POINT.

A._____ Kronecker, Poincaré and Cantor were all supporters of the idea that multiple sizes of infinity exist.

B._____ Gauss and Cauchy both have probability density functions named after them.

C._____ Euler is known as the "Prince of Mathematicians."

D._____ Weierstrass and Abel were on opposite sides of the effort to strengthen mathematical rigor in the nineteenth century.

E._____ The Basel Problem and the Konigsberg Bridge Problem were both solved by Euler.