History of Mathematics

Math 395 Spring 2010 Fowler 310 MWF 10:30am - 11:25am
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Class 29: **Monday April 19**

TITLE Gauss: Prince of Mathematicians
CURRENT READING: Katz, §21
NEXT READING: Katz, §22

Homework #10 on Friday April 23rd
Katz, page 636-639: 2,17, 18, 39. EXTRA CREDIT: 32, 38.

SUMMARY
The last great mathematical genius we will be considering is Carl Friedrich Gauss.

Carl Friedrich Gauss (1777-1855)
Gauss gets his nickname as the "Prince of Mathematicians" from the inscription "*Princeps mathematicorum*" that appeared on his tombstone. Gauss is one of the most precocious child prodigies of all time, with stories abounding of his mathematical prowess from as early as age 3. he himself is said to have said "I could figure before I could talk" (Ball).

Proving the Fundamental Theorem of Algebra
Gauss' Ph.D. thesis at the age of 20 contained the first satisfactory of the fundamental theorem of algebra (i.e. That a polynomial equation with complex coefficients of degree $n>0$ must have at least one root in the complex plane). Gauss returned to this theorem over and over again in his life, publishing two more rigorous proofs in 1816 and another (#4!) in 1850.

The main idea of Gauss' first proof is that a polynomial $f(z)=0$ where $z=a+ib$ is a complex variable will really turn into two polynomials $r(a,b)=0$ and $c(a,b)=0$ by isolating the real and complex parts of the equation.

Exercise

Show that a polynomial of the form $x^2 + 4i = 0$ has a root of the form $a+bi$.

Number Theory

Disquisitiones arithmeticae (Investigations in Arithmetic) is Gauss' masterwork in number theory, published in 1801. Gauss called Mathematics the "queen of the sciences, and the theory of numbers if the queen of mathematics" (Eves). It was written in Latin, with a French translation appearing in 1807 but an English version was unavailable until 1966(!)

Investigations in Arithmetic begins with the definition:

> **If a number *a* divides the difference between two numbers *b* and *c*, then *b* and *c* are said to be congruent, other incongruent; and *a* itself is called the modulus. Either of the numbers is called the residue of the other, in the former case, a nonresidue in the latter case.**

The terms and notation Gauss introduced in this book are still used today. i.e.

$$b \equiv c(\mathrm{mod}\, a)$$

Gauss then went on to create a new form of algebra using congruency as a form of equality.

Exercise

Let us find solutions of the congruence $3x \equiv 51(\mathrm{mod}\, 9)$. You should be able to find three less than 9.

Carl and Sophie

Sophie Germain (1776-1831) was able to master Calculus on her own and read Gauss's *Disquisitiones* when it came out and wrote the author letters under the pseudonym M. Le Blanc, to which he replied. However, due to her gender she was barred from attending the École Polytechnique in Paris. However, lecture notes were distributed freely to anyone who asked and Germain was able to study via letters to Lagrange and Legendre, who both taught there. Eventually, Lagrange became curious about M. LeBlanc and asked for a meeting, but was unperturbed to discover that his correspondent was female. Germain also inadvertently revealed her identity to Gauss, because fearing for an "Archimedean accident" when the French military was occupying the city that Gauss was living in, she wrote to the general in charge who personally saw to it that Gauss was safe, but was then confused as to how Mme. Germain could know of his existence. Gauss, too was pleased to discover M. Le Blanc was a female and according to Katz, wrote:

> **When a person of the sex which, according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents, and a superior genius.**

Gauss' Other Contributions

Gauss made contributions to Astronomy (producing an accurate prediction of the orbit of the "planet" Ceres using a technique which is still used today to track satellites called Gauss's Method) as well as statistics and geophysics.