History of Mathematics

Math 395 Spring 2010 Fowler 310 MWF 10:30am - 11:25am

http://faculty.oxy.edu/ron/math/395/10/

Class 26: **Friday April 9**

TITLE 18th Century Adoption and Promotion of Calculus: The Bernouillis
CURRENT READING: Katz, §17.3, §16.3
NEXT READING: Katz, §17.1-17.2

Homework #9 DUE Monday April 12 (in class)
Katz, p. 539: #1, #6, #11, #25, #26. EXTRA CREDIT: page 540, #18, #19.

SUMMARY
We will discuss the work of the people who popularized The Calculus in the early 18th Century.

Jakob Bernouilli (1654-1705) and **Johann Bernouilli** (1667-1748) were two of the first adopters of the Leibnizian calculus and used it to solve important problems in the early 18th century. Jakob and Johann (sometimes called James and John) are the most famous brothers in the history of mathematics.

According to Boyer, the Jakob and Johann's father Nicolaus (1623-1708) had selected career paths for his son which did not include them being mathematicians. Jakob was supposed go into the ministry and Johann a merchant or physician.

The First Calculus Text and "l'Hospital's Rule"
Perhaps because of this Johann made an unusual arrangement with **Marquis Guillaume François Antoine L'Hospital** (1661-1704) in which Johann would be paid a salary and in return send L'Hospital his mathematical discoveries.

L'Hospital published the very first textbook on differential calculus in Paris in1696, called *Analyse des infiniment petits pour l'intelligence des Lignes courbes (Analysis of the Infinitely Small For The Understanding of Curves)* and in it he included a result of John Bernouilli's which has since then become incorrectly known as "L'Hospital's Rule."

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g('x)}$$

Maria Agnesi (1718-1799)
The second most important European calculus text was published by Maria Agnesi in 1748 in Italian *Instituzioni analitiche ad usodella gioventu italiana* (*Foundations of Analysis for the Use of Italian Youth*) with a French edition in 1749. Agnesi is "the first important woman mathematician since Hypatia" (Struik). Unsurprisingly, Agnesi used Leibniz' notation, although the English translation in 1801(!) replaced all *dx*'s by $\dot{x}$'s (Katz 616).

British Calculus texts
In England, mathematicians insisted on using the notation and monikers derived by Newton. **Thomas Simpson** (1710-1761) published *A New Treatise on Fluxions* in 1737 and **Colin Maclaurin** (1698-1746) published *a Treatise on Fluxions* in 1742.

The Critic

George Berkeley (1685-1743) was an Irish philosopher who printed a harsh criticism of Newton's calculus in 1734 which was titled (deep breath): *The Analyst, Or a Discourse to an Infidel Mathematician.* The subtitle was:

> ***Wherein It Is Examined Whether the Object, Principles, and Inferences of the Modern Analysis are More Distinctly Conceived, or More Evidently Deduced, than Religious Mysteries and Points of Faith. "First Cast the Beam Out of Thine Own Eye; And then Shalt Thou See Clearly to Cast Out the Mote of Thy Brother's Eye."***

Berkeley pointed out that Newton's fluxional calculus was useful at solving problems but said "by virtue of a twofold mistake you arrive, though not at science, yet at the truth" and scoffed at the notion of an instantaneous velocity which results from the quotient 0/0. The infidel mathematician in the title is widely believed to be Edmund Halley not Newton himself.

MacLaurin's text was an attempt to place the Calculus in a firmer context which he did by using more geometrically based techniques and he is widely considered the most talented British mathematician of the 18th century (Ball and Boyer).

MacLaurin is now best known for the Maclaurin Series where he assumes that a fluent y can be expressed as a power series in z the coefficients will be fluxions.

$$y = E + \dot{E}z + \frac{\ddot{E}z^2}{1\times 2} + \frac{\dddot{E}z^3}{1\times 2\times 3} + \ldots$$

Even he noted that the result had been published in a work by **Brook Taylor** (1685-1731) in his *Methodus incrementorum (Method of Increments)* in 1715.

The Bernouilli brothers were renowned for their applications of calculus to practical problems, especially in the nascent area of calculus of variations.

The Catenary

The shape of the curve of a flexible but inelastic cord hung between two points is known as **the catenary**. Jakob Bernouilli was unable to solve this problem but Huygens and Leibniz and (horrors!) his brother Johann were.

The solution in diffrential form is

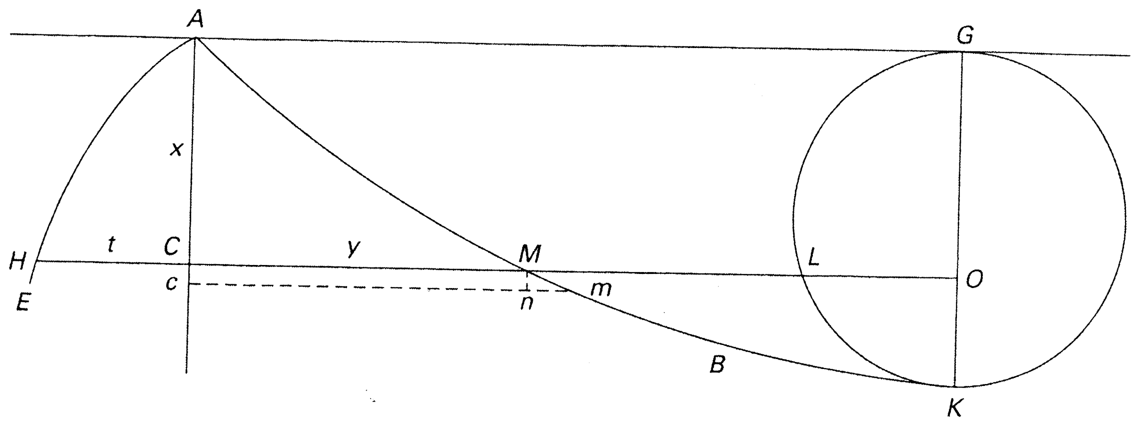
$$dx = \frac{a\,dy}{\sqrt{y^2 - a^2}}$$

Exercise

Let's confirm that the closed form of the catenary is that $x = a\ln(y + \sqrt{y^2 - a^2})$ or $y = a\cosh(x/a)$

The Brachistone Problem

Johann Bernouilli is famous for his proposition and solution of the brachistone problem, also known as the curve of quickest descent. As Katz notes, in the June 1696 issue of Leibniz's journal *Acta eruditorum*: "If two points *A* and *B* are given in a vertical plane, to assign to a mobile particle *M* the path *AMB* along which, descending under its own weight, it passes from the point *A* to point *B* in the briefest time."



Bernouilli's solution involved deriving the following from the idea that sine of the angle nMm is proposrtional to the velocity t of the particle, $\sin nMm \propto t$

$$dy : t = ds : a$$

$$a\,dy = t\,ds$$

$$a^2 dy^2 = t^2 ds^2$$

$$a^2 dy^2 = t^2 dx^2 + t^2 dy^2$$

Which can be solved to produce

$$dy = \frac{t\,dx}{\sqrt{a^2 - t^2}}$$

$$dy = dx\sqrt{\frac{x}{a-x}}$$

$$dx\sqrt{\frac{x}{a-x}} = \frac{a\,dx}{2\sqrt{ax-x^2}} - \frac{(a-2x)\,dx}{2\sqrt{ax-x^2}}$$

Given that $y^2 = ax - x^2$ we can show that this curve is the cycloid, which is usually given parametrically by the equations $x = a(t - \sin t), y = a(1 - \cos t)$

GroupWork

Let's confirm that the Bernouilli's solution for the brachistrone curve is indeed a cycloid.