# History of Mathematics

Math 395 Spring 2010 Fowler 310 MWF 10:30am - 11:25am
 http://faculty.oxy.edu/ron/math/395/10/

*Class 19* :**Wednesday March 22**

**TITLE** Mathematics of the Renaissance Period
**CURRENT READING:** Katz, §12
**NEXT READING:** Katz, §13

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**Homework #8 DUE Friday April 2 (in class)**
**Katz, p. 418: #4, #8, #21, #30, #37. EXTRA CREDIT: page 419, #29.**

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**SUMMARY**
We will look at the mathematical developments of the early Renaissance period.

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**Cardano's Solution of the Cubic Equation**
**Gerolamo Cardano** (1501-1576) was the first person to publish a solution to the solution of the cubic equation $x^3 + cx = d$ in his *Ars magna, sive de regulis algebraicis* (*The Great Art, or On The Rules of Algebra*).

Cardano's Formula is

$$x = \sqrt[3]{\sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{c}{3}\right)^3} + \frac{d}{2}} - \sqrt[3]{\sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{c}{3}\right)^3} - \frac{d}{2}}$$

**Nicolo Tartaglia** (1499-1557) had first shown that if you select two numbers *u* and *v* such that $u^3 - v^3 = d$ and $u^3v^3 = \left(\frac{c}{3}\right)^3$ then the solution to $x^3 + cx = d$ is $x = u - v$.

**Exercise**
Let's use Cardano's formula to solve the cubic $x^3 + 6x = 20$

**EXAMPLE**
Let's show that Tartaglia's conditions on *u* and *v* do indeed solve $x^3 + cx = d$

**GroupWork**

We should be able to derive Cardano's formula considering that Tartaglia's conditions correspond to solving a system of equations for two unknown quantities whose difference and product are known quantities.

**First**, we can show that $py^3 + qy^2 + ry = s$ can be made to look like $x^3 + cx = d$ by dividing by $p$, and using the transformation $y=x+\beta$ and selecting $\beta = -\frac{1}{3}\frac{q}{p}$

**Second**, solve the system $a\text{-}b = x$ and $ab = \frac{1}{3}c$ and re-derive Cardano's formula.

**Notation issues**

| | |
|---|---|
| *cosa* | thing |
| *censo* | square |
| *cubo* | cube |
| *radice* | root |
| $\bar{p}$ | più (plus) |
| $\bar{m}$ | meno (minus) |

Note, that Cardano did not use modern notation but would have written down the solution to the solution to $x^3 + 6x = 20$ as

$$\mathcal{R}\ \text{v : cub}\ \mathcal{R}\ 108\ \text{p : }10\ \text{m : }\mathcal{R}\ \text{v : cub}\ \mathcal{R}\ 108\ \text{m : }10$$

Generally, this kind of algebraic manipulation is called **rhetorical** as opposed to **symbolic** which came later.

**Don't Fear The Square Root!**
Look at this problem by Antonio de Mazzinhi (1353-1383): "Find two numbers such that multiplying one by another makes 8 and the sum of their squares is 27." **Ans:** $x = \frac{\sqrt{43}}{2}, y = \frac{11}{4}$

The solution involves choosing the first number is *un cosa meno la radice d'alchuna quantità* (a thing minus the root of some quantity) while the second number equals *una cosa più la radice d'alchuna quantità* (a thing plus the root of some quantity)

**Exercise**
Use Mazzinhi's method to solve the above problem.

**The Beginning of Imaginary Numbers**
Cardano also gave a formula for the solution of $x^3 = cx + d$, namely

$$x = \sqrt[3]{\frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{c}{3}\right)^3}} + \sqrt[3]{\frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{c}{3}\right)^3}}$$

**Rafael Bombelli** (1526-1572) learned how to deal with examples of Cardano's formula for the cubic $x^3 = cx + d$ where the root becomes complex because $\left(\frac{d}{2}\right)^2 - \left(\frac{c}{3}\right)^3$ becomes negative. According to Katz, Bombelli proposed a name for such numbers as "neither positive (*più*) nor negative (*meno*)." What we call imaginary numbers, such as *bi* and –*bi*, Bombelli called *più di meno* (plus of minus) and *meno di meno* (minus of minus), respectively.

Bombelli gave multiplication rules for these new numbers, such as:

*più di meno* times *più di meno* equals *meno* and *meno di meno* times *più di meno* equals *più*
**In Modern Notation:**

**Practical Uses**

Bombelli was able to show that the solution to $x^3 = 15x + 4$ is $x$=4, even though by Cardano's formula one should get

$$x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}}$$

However, if one assumes

$$\sqrt[3]{2+\sqrt{-121}} = a + \sqrt{-b}$$

$$\sqrt[3]{2-\sqrt{-121}} = a - \sqrt{-b}$$

One can obtain the equations $a^2+b=5$ and $a^3-3ab=2$ which Bombelli carefully showed has the solution $a=2$ and $b=1$.

Using this information, we can obtain the solution to the cubic to be $x$=4. Bombelli was able to use this knowledge to solve previously "unsolvable" quadratic equations like $x^2 + 20x = 4$

***The Analytic Art***

**François Viète** (1540-1603) developed theories for solving problems based on the work of the Greeks and published a work called *In artem analyticem isagoge* (*Introduction to the Analytic Art*) in 1591. Pappus had divided analysis into two parts: "problematic analysis" and "theorematic analysis."

Viète renamed these kinds of analysis and added a third.

Problematic analysis became **zetetic analysis** (the procedure by which one transforms a problem into an equation linking the unknown and various knowns).

Theorematic analysis became **poristic analysis** (the procedure exploring the truth of a theorem by appropriate symbolic manipulation)

**Exegetics** is the art of transforming an equation found by zetetic analysis to find a value for the unknown.

Viète is also known for his introduction of new symbols to represent terms, and was one of the first people to do algebra in a symbolic, not rhetorical manner.

Viète would write the equation $A^2 + 2BA = Z$ as "A quad + B2 in A equals Z plane" and its solution as

$A = \sqrt{Z^2 + B^2} - B$ becomes A is $l.\overline{Z\text{plane} + B\text{quad}} - B$ which Katz records as the first occurrence of the quadratic formula as we understand it today, in symbolic form.

Viète wrote Cardano's formula for the equation "A cube – B plane 3 in A equals Z solid 2" as

A is $l.c.\overline{Z\text{solid} + l.\overline{Z\text{ solidsolid} - B\text{ planeplaneplane}}} + l.c.\overline{Z\text{solid} - l.\overline{Z\text{ solidsolid} - B\text{ planeplanepla}}}$