# History of Mathematics





*Class 16*: **Friday March 5**

**TITLE** Islamic Mathematics
**CURRENT READING:** Katz, §9
**NEXT READING:** Katz, §10

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**Homework #6 DUE Monday March 15 (5pm)**
**Katz, p. 226: #2,#14,#15. p.260: 9, 12. EXTRA CREDIT: page 226, #9.**

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**SUMMARY**
We will begin to explore the contributions of the Islamic world to preserve and contribute to mathematical knowledge.

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**Islamic versus Arabic**
The word Islamic is used to describe the contributions that occurred in the vast geographic region that was controlled by Muslims at one time.

**Islamic Era**
The Islamic era dates from around 622 CE when Mohammed left Mecca to go to Medina, often called "the Hejira." Mohammed died suddenly in Medina in 632 CE but this did not stop the rapid military expansion of Moslem forces to control an area as far west as Spain and as far east as India and parts of central Asia.

In 766, the caliph al-Mansur founded his capital at Baghdad, which became a commercial and intellectual center of the Arabian empire. The caliph al-Rashid established a library in Baghdad and began a program of collecting Greek manuscripts from the cities of Athens and Alexandria and translating them into Arabic. His successor, caliph al-Ma'mun, established the *Bayt al-hikma* (House of Wisdom) in Baghdad in an attempt to replicate the ancient research center at Alexandria.

**The Father of Algebra**
Mohammed ibn Musa al-Khwarizmi (c. 750-850) is the most famous of the Arabic mathematicians and is sometimes called the "father of algebra."

His work is so influential that he is credited with coining two words: algorithm and algebra. The word "algorithm" comes from a Latin description of al-Khwarizmi's work was described as "Dixit Algorismi" which became associated with doing arithmetic operations and turned into the English word algorithm.

The word "algebra" comes from "al-jabr" which appeared in the title of al-Khwarizmi most famous work *Al-kitab al-muhtasar fi hisab* ***al-jabr*** *wa-l-muqabala* (The Condensed Book on the Calculation of *al-Jabr* and *al-Muqabala*). *Al-jabr* was generally understood to refer to the operation of transposing a term from one side of the equation to another and *al-muqabala* is generally understood to mean comparing terms.

## The work of al-Khwarizmi

In *The CondensedBook on the Calculation of al-Jabr and al-Muqabala* al-Khwarizmi systematically showed how to solve the kinds of equations which involved the square, the root of the square and the absolute number. There are six such kinds of equations

(Squares are equal to roots) $ax^2 = bx$
(Squares are equal to numbers) $ax^2 = c$
(Roots are equal to numbers) $bx = c$
(Squares and roots are equal to numbers) $ax^2 + bx = c$
(Squares and numbers are equal to roots) $ax^2 + c = bx$
(Squares are equal to roots and numbers) $ax^2 = bx + c$

Note that all the coefficients are positive and that zero was not a solution allowed by al-Khwarizmi, since his technique was basically geometric (like the Babylonians and Greeks).

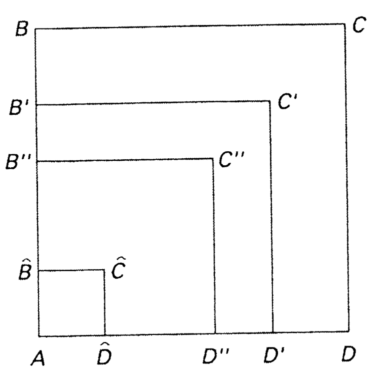
**GroupWork**

Write down the algebraic solution to all 6 types of al-Khwarzimi's equations. How many different problems would we classify these into today?

### The Birth of Proof by Induction

Abu Bakr al-Karaji (d. 1019) gave the following result in the first decade of the 11th century in his book entitled *al-Fakhri* (*The Marvelous*)

$$1^3+2^3+3^3+4^3+\ldots+10^3=(1+2+3+4+..+10)^2$$




The square ABCD has side 1+2+3+..+10

The gnomon BCDD'C'B' has area $10^3$

Area ABCD = Area AB'C'D' + Area Gnomon BCDD'C'B'

$(1+2+\ldots.10)^2 = (1+2+3+\ldots+10)^2+10^3$

Repeat the process with the next smaller square and gnomon

Finally the smallest square ABCD = smallest gnomon ÂBCD since $1=1^3$

How is this process (similar) different from what we know as mathematical induction today?

**General Relation**

Egyptiam mathematician Abu Ali al-Hasan ibn al-Hasan ibn al-Haytham (965-1039) derived the equation

$$(n+1)\sum_{i=1}^{n} i^k = \sum_{i=1}^{n} i^{k+1} + \sum_{p=1}^{n}\left(\sum_{i=1}^{p} i^k\right)$$

Ibn al-Haytham (also known as Alhazen) did not give the general form but for particular integers $n$=4 *and* k=1,2 3.

**GroupWork**

Let's show how we can use these formulas to generate the following reasonably well-known formulas:

$$\sum_{i=1}^{n} i^2 = \left(\frac{n}{3}+\frac{1}{3}\right) n \left(n+\frac{1}{2}\right) = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n}{4}+\frac{1}{4}\right) n(n+1)n = \frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}$$

$$\sum_{i=1}^{n} i^4 = \left(\frac{n}{5}+\frac{1}{5}\right) n \left(n+\frac{1}{2}\right)\left[(n+1)n-\frac{1}{3}\right]$$