History of Mathematics

Math 395 Spring 2010 — Fowler 310 MWF 10:30am - 11:25am

 — http://faculty.oxy.edu/ron/math/395/10/

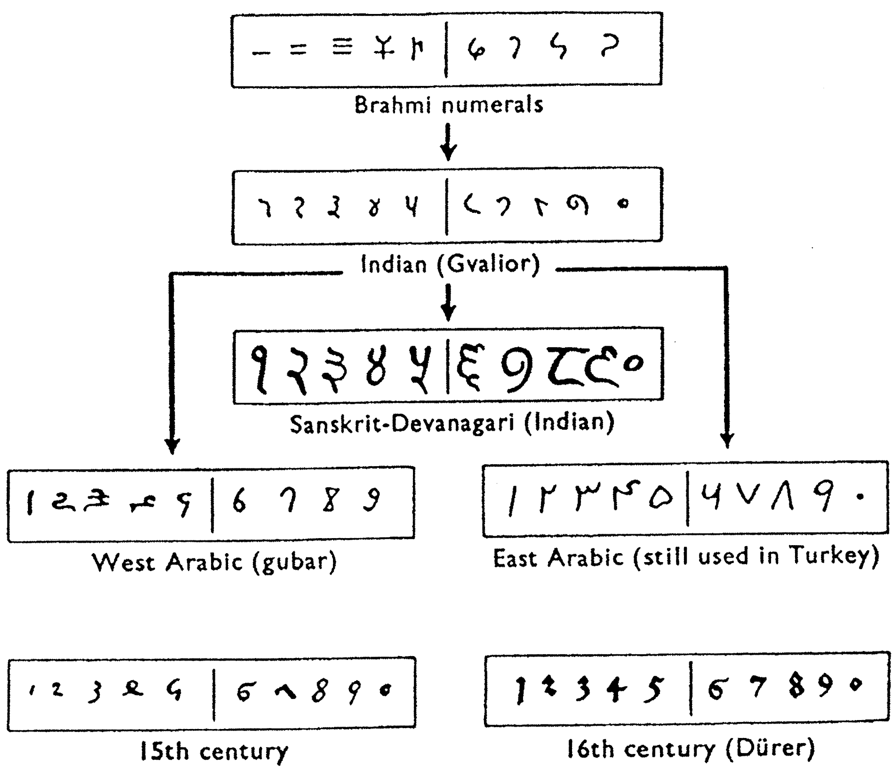
Class 15: **Wednesday March 3**

TITLE Indian Mathematics
CURRENT READING: Katz, §8.1-8.8
NEXT READING: Katz, §9

Homework #6 DUE Monday March 15 (5pm)
Katz, p. 226: #2,#14,#15. p.260: 9, 12. EXTRA CREDIT: page 226, #9.

SUMMARY
We will examine the mathematics and mathematicians of the Indian "sub-continent" in the ancient era, from which our modern representations of numbers was developed.

Modern Numerals and Decimal Place System
The Indians are most well-known for first using only 10 symbols combined with a place-value system to represent numbers of all magnitudes. They also popularized the use of a symbol to represent zero.



Katz also mentions that Indians used words to represent individuals numerals as well, such as sky for 0, moon for 1, eye for 2, fire for 3. They used a place system with units starting at the left.

EXAMPLE
moon-eye-sky-fire would be 3021. What would 2003 be?

The significance of the Indian contribution to the way we represent numbers today has often not been recognized but should not be forgotten. An eminent French mathematician, Pierre-Simon Laplace, said:

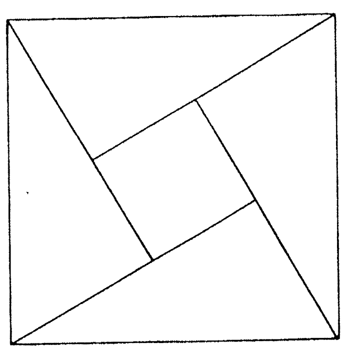
> It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity (Eves, 1988).

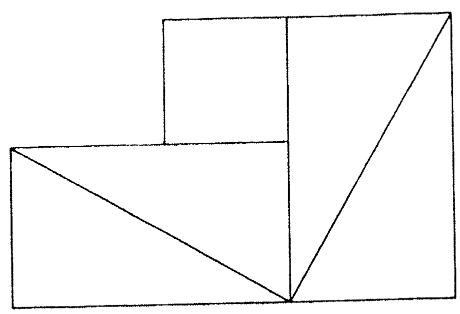
It should be noted that the Indians did not use decimal fractions, that was developed by the Islamic mathematicians.

Brahmagupta and **Bhaskara** are two of the most famous Indian mathematicians. They both flourished in the 7th century CE. There was a second mathematician with the name Bhaskara later

Bhaskara I's Proofs of Pythagoras' Theorem

Bhaskara gave a pictorial "proof" of the Pythagorean theorem (which had clearly already been known for hundreds of years in India at the time because it appeared in older Indian writings called the *Sulbasutras.* He gave the following pictures and simply wrote "Behold!" (Eves, 1990).





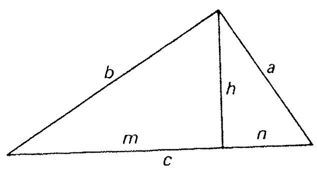
EXAMPLE

Let's show that given the sides of the triangle are (shorter side)*a* and (longer side) *b with hypotenuse* c, Bhaskara's proof is equivalent to showing that

$$c^2 = 4\left(\frac{ab}{2}\right) + (b - a)^2 = a^2 + b^2$$

Exercise

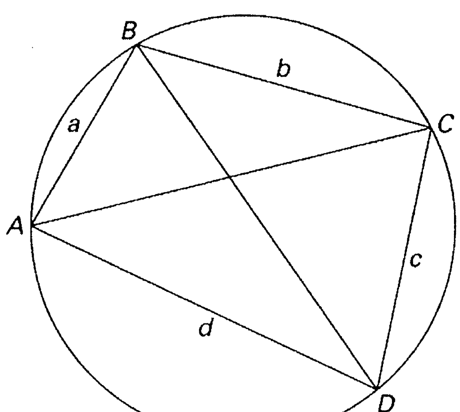
Bhaskara's 2nd Proof. First prove (all) the triangles are similar to obtain the expressions below, and then use that to show $c^2 = a^2 + b^2$



$$\frac{c}{b} = \frac{b}{m}, \quad \frac{c}{a} = \frac{a}{n}$$

The Brahmagupta trapezium

Brahmagupta gives the following amazing result in his *Brahmaphutasiddhanta* (*Correct Astronomical System of Brahma*). Heron's formula for the area of a triangle is a special case of this result. **Recall Ptolemy's Theorem:** $ac+bd=|AC||BD|$



$$AC = \sqrt{\frac{(ac+bd)(ad+bc)}{ab+cd}}$$

$$BD = \sqrt{\frac{(ac+bd)(ab+cd)}{ad+bc}}$$

$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
where
$s = \frac{1}{2}(a+b+c+d)$

The Etymology of "Sine"

Katz points out that the modern word "sine" is a result of incorrect translations of the word *jya-ardha* from Sanskrit, which means "chord-half." Aryhabhata abbreviated the term to *jya* or *jiva* which when translated into Arabic became *jiba* (which is not a word in Arabic). However, since Arabic is written without vowels, later Arabic readers saw the letters *jb* and assumed that it was representing the Arabic word *jaib* which means bosom or breast. Then, when the Arabic was translated into Latin in the 12th century the Latin word *sinus* was used (which means bosom). It was the Latin word *sinus* which became our modern English word **sine**!

Interestingly, Indians knew power series approximations of several trigonometric functions, like

$$\cos s \approx 1 - \frac{s^2}{2} + \frac{s^4}{24} \qquad \sin s \approx s - \frac{s^3}{6} + \frac{s^5}{120}$$

Aryabhata (b. 476)
He is one of the earliest identifiable Indian mathematicians and wrote a book of mathematical results called *Aryabhatiya* where it is clear that he was able to apply the quadratic formula.

STANZA II, 19 *The desired number of terms minus one, halved, . . . multiplied by the common difference between the terms, plus the first term, is the middle term. This multiplied by the number of terms desired is the sum of the desired number of terms. Or the sum of the first and last terms is multiplied by half the number of terms.*[15]

This corresponds to the formula

$$S_n = n\left[\left(\frac{n-1}{2}\right)d + a\right] = \frac{n}{2}[a + (a + (n-1)d)].$$

where S_n is the sum of an arithmetic progression with constant difference d and first term a.

STANZA II, 20 *Multiply the sum of the progression by eight times the common difference, add the square of the difference between twice the first term and the common difference, take the square root of this, subtract twice the first term, divide by the common difference, add one, divide by two. The result will be the number of terms.*

In other words, by considering the formula for S_n given in Stanza II, 19 as a quadratic in n we can show that it can be solved for n.

$$n = \frac{1}{2}\left[\frac{\sqrt{8S_n d + (2a-d)^2} - 2a}{d} + 1\right]$$

GroupWork

Consider the sequence of numbers a, $a+d$, $a+2d$, $a+3d$,.... What is the n^{th} term? Obtain a formula for the sum of the first n terms and confirm that your formula matches Aryabhata's given in Stanza II, 19.

Then use the quadratic formula on your answer to obtain an expression for the number of terms it takes to reach a fixed sum S_n and confirm that your formula matches Aryabhata's given in Stanza II, 20.