History of Mathematics

Math 395 Spring 2010
Fowler 310 MWF 10:30am - 11:25am

http://faculty.oxy.edu/ron/math/395/10/

Class 14: **Wednesday February 24**

TITLE Chinese Mathematics, continued
CURRENT READING: Katz, §7
NEXT READING: Katz, §8

Homework #5 DUE Monday March 1
Katz, p. 168: #2,#20. p.191: 7, 11, 21. EXTRA CREDIT: page 168, #4.

SUMMARY
We will look at two famous Chinese problems: the Chinese Remainder problem and 'the 100 fowls" problem.

Simultaneous linear congruences
In *Mathematical Classic of Master Sun* (*Sunzi suanjing*) from 300 CE the following problem appears:

> ***We have things of which we do not know the number; if we count them by threes, the remainder is 2; if we count them by fives, the remainder is 3; if we count them by sevens, the remainder is 2. How many things are there?***

In modern notation, this becomes a problem of simultaneous linear congruences:

Find N, such that

$$N \equiv 2 \pmod 3 \quad N \equiv 3 \pmod 5 \quad N \equiv 2 \pmod 7 \qquad (1)$$

The answer is $N=23$.

Katz reports Sun Zi's solution:

> "If you count by threes and have the remainder 2, put 140. If you count by fives and have the remainder 3, put 63. If you count by sevens and have the remainder 2, put 30. Add these numbers and you get 233. From this subtract 210 and you get 23."

It turns out that

$$70 \equiv 1 \pmod 3 \equiv 0 \pmod 5 \equiv 0 \pmod 7$$
$$21 \equiv 1 \pmod 5 \equiv 0 \pmod 3 \equiv 0 \pmod 7$$
$$15 \equiv 1 \pmod 7 \equiv 0 \pmod 3 \equiv 0 \pmod 2$$

So, if you want to find N which satisfies all three equations in (1) simultaneously it can be computed as

$$N=70 \times 2 + 21 \times 3 + 15 \times 2 = 140+ 63+30 = 233=23 \pmod{105}$$

The modern Chinese Remainder Theorem is the generalized version of the Sun Zi problem.

The Hundred Fowls Problem

In *Mathematical Classic of Zhang Quijian*from the 5th century CE the hundred flows problem appears:

"A rooster is worth 5 coins, a hen 3 coins, and 3 chicks 1 coin. With 100 coins we buy 100 of the fowls. How many roosters, hens and chicks are there?"

This corresponds to the system of two equations in three unknowns:

$$5x+3y+\frac{1}{3}z=100$$
$$x+y+z=100$$

Zhang gave three answers: "4 roosters, 18 hens, 78 chicks; 8 roosters, 11 hens, 81 chicks; 12 roosters, 4 hens, 84 chicks."

GroupWork

Show that the general solution of this system is x=-100+4t, y=200-7t, z=3t.