History of Mathematics

Math 395 Spring 2010 — Fowler 310 MWF 10:30am - 11:25am

 — http://faculty.oxy.edu/ron/math/395/10/

Class 12: **Friday February 19**

TITLE Diophantus to Hypatia: The Beginning of Algebra to the End of The Greek Era
CURRENT READING: Katz, §6
NEXT READING: Katz, §7

Homework #5 DUE Monday March 1
Katz, p. 168: #2,#20. p.191: 7, 11, 21. EXTRA CREDIT: page 168, #4.

SUMMARY
Today we will review some of the work of Diophantus, who introduced the use of symbols to abbreviate and represent terms in equations and be introduced to the earliest known female mathematician, Hypatia of Alexandria (daughter of Theon).

Symbolism of Diophantus
He used various symbols to represent the unknown (what we call x)

$$x = \begin{cases} \zeta \quad S \quad \zeta' \\ y \qquad 'S^{\circ} \quad \alpha\rho \end{cases}$$

Unknown powers of x were denoted using Δ and/or K.

$$\begin{aligned} x^2 &:= \Delta^{\Upsilon} \\ x^3 &:= K^{\Upsilon} \\ x^4 &:= \Delta^{\Upsilon}\Delta \\ x^5 &:= \Delta K^{\Upsilon} \\ x^6 &:= K^{\Upsilon}K \end{aligned}$$

There was no equivalent symbol to + for addition. The default operation (no space between symbols) represented addition.

$$\pitchfork := \text{minus} \qquad \overset{\circ}{M} := \text{units}$$

These symbols would be combined to represent what we know as a polynomial now.

EXAMPLE

$$K^{\Upsilon}\alpha\,\Delta^{\Upsilon}\iota\gamma\,\zeta\varepsilon\,\overset{\circ}{M}\beta = x^3 + 13x^3 + 5x + 2$$

What algebraic expression do these collections of symbols on the left represent?

$$K^{\Upsilon}\alpha\,\zeta\eta\,\pitchfork\Delta^{\Upsilon}\varepsilon\,\overset{\circ}{M}\alpha$$

$$\Delta^{\Upsilon}\iota\varepsilon\,\pitchfork\overset{\circ}{M}\,\lambda\theta$$

symbol	value	symbol	value	symbol	value
α	1	ι	10	ρ	100
β	2	κ	20	σ	200
γ	3	λ	30	τ	300
δ	4	μ	40	υ	400
ϵ	5	ν	50	ϕ	500
ς	6	ξ	60	χ	600
ζ	7	o	70	ψ	700
η	8	π	80	ω	800
θ	9	ϙ	90	ϡ	900

Exercise

Write $x^2 - 2x + 1$ using Diophantine notation.

Make up your own polynomial and trade with your nearest neighbor!
Modern:

Diophantine:

Diophantine equations

Diophantus only allowed solutions which were positive integers. Equations which satisfy this property are called **Diophantine equations**.

Diophantus generally solved two different kinds of equations in his most well-known work, called *Arithmetica*: "determinate equations" (single variable) and indeterminate equations (two or more unknowns in a single equations)

EXAMPLE

Problem II-8: ***To divide a given square number into two squares.***

Let $x^2 + y^2 = b^2$ where b^2 is "the given square."
Take any value for a and let $y=ax-b$.

Show that $x = \dfrac{2ab}{a^2+1}$ and $y = \dfrac{b(a^2-1)}{a^2+1}$ solves the problem.

GroupWork

Problem IV-31: ***To divide unity into two parts so that, if given numbers are added to them, respectively, the product of the two sums is a square.***

In modern symbols this problem can be represented algebraically as: $(x+a)(1-x+b)=y^2$

Note: that this problem has _____ variable(s) and _________ parameter(s).

Diophantus chose a=3 and b=5 and showed the solutions are then $\frac{6}{25}$ and $\frac{19}{25}$

Pappus of Alexandria (fl. 300 BCE)
Pappus was considered the last great Greek mathematician. By this time much of the amazing works of the Greeks had been lost and he wrote his *Collection* to try and restore that knowledge.

Hypatia of Alexandria (c. 355-415 CE)
Hypatia was well-known as a commenter on the works of earlier Greek mathematicians such as Ptolemy, Archimedes, Apollonius, Diophantus and Pappus. She was also well-known as the daughter of Theon of Alexandria. Her death at the hands of an angry mob who equated her mathematical knowledge and expertise with witchcraft is often the most widely known fact about her.