History of Mathematics

Math 395 Spring 2010 | Fowler 310 MWF 10:30am - 11:25am

http://faculty.oxy.edu/ron/math/395/10/

*Class 11*: **Wednesday February 17**

**TITLE** Ptolemy and the dawn of trigonometry
**CURRENT READING:** Katz, §5.1-5.3
**CURRENT READING:** Katz, §6

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**Homework #4 for Friday February 19**
**Katz, p. 127-129. #1,#4,#12,#17, #18 and #34. EXTRA CREDIT: #5.**

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**SUMMARY**
Claudius Ptolemy (CE 100-178) is most well-known for his model of the solar system and his publication of *The Almagest.*

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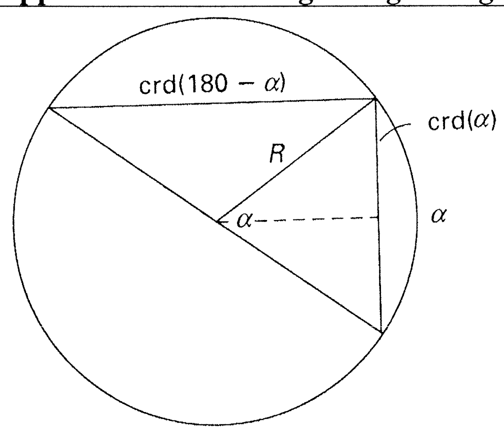
**Hipparchus and the beginning of trigonometry**




$$\frac{1}{2}\,\mathrm{crd}(\alpha)/R = \sin\frac{\alpha}{2}$$

$$\mathrm{crd}(\alpha) = 2R\sin\frac{\alpha}{2}$$

$$crd(180-\alpha) = \sqrt{(2R)^2 - (crd(\alpha))^2}$$
$$= 2R\cos(\frac{\alpha}{2})$$

$$crd^2(\frac{\alpha}{2}) = R(2R - crd(180-\alpha))$$

**Hipparchus of Bythnia** (190-120 BCE) was defined the length of a chord subtended by an angle α, denoted chord(α) or crd(α) by Katz. This marked the beginning of trigonometry as we know it today.

Hipparchus constructed a table of chords and used it to make astronomical calculation of surprising accuracy. He used a sexagesimal approximation of π to be 3;8,30 and assuming that there were 6,0,0 minutes (360 degrees divided into 60 minutes) in a circle he computed that a radius of a circle had to be 3438 minutes long, or 57,18 (in sexagesimal).

He calculated the length of the solar year to be 365 ¼ days, less 4 minutes, 48 seconds (off by 6 minutes from modern calculations) and the length of the lunar month to be 29 days, 12 hours, 44 minutes, 2 ½ seconds (less than 1 second off). Source: G. Donald Allen's *Ancient Greek Mathematics*.

Hipparchus' work was exceeded by the work of Claudius Ptolemy, who produced a table of chords from every angles from one-half a degree up to 180 degrees (in sexagesimal, of course). See Table 5.1 of Katz.

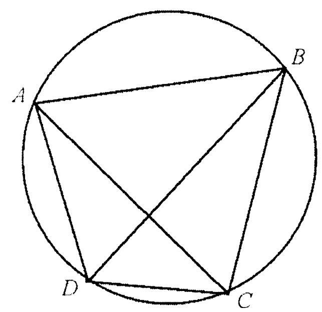
***The Almagest***

Ptolemy published *Mathematical Collection* (*Mathematiki Syntaxis*) which was translated into Arabic and because it was the predominant astronomical work for centuries it became known as *megisti syntaxis* (the greatest collection) or "*al-magisti*" or in English, the *Almagest*.

Almost nothing is known about Ptolemy's personal life but he developed a mathematical model which described the motion of the sun, moon and known planets.

**Ptolemy's Theorem**: *Given any quadrilateral inscribed in a circle, the product of the diagonals equals the sum of the products of the opposite sides.*

**Theorem.** $|AC|\,|BD| = |AD|\,|BC| + |AB|\,|DC|$



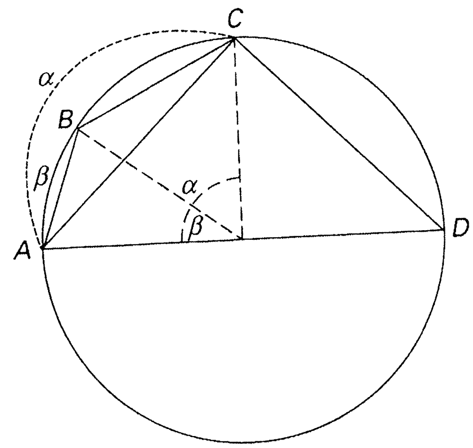

Ptolemy's Theorem $|AC|\,|BD| = |AD|\,|BC| + |AB|\,|DC|$

**EXAMPLE**

Let's try and work through the proof of the theorem (Katz, page 147).

**Applying Ptolemy's Theorem**

We can reproduce some trigonometric identities. Consider the figure below (Katz, Figure 5.18):




It turns out that letting AD=crd(α) and AB=crd(β) then BC=crd(α- β).
Applying Ptolemy's Theorem to the quadrilateral ABCD produces:

$$120 \,\mathrm{crd}(\alpha-\beta) = \mathrm{crd}(\alpha)\,\mathrm{crd}(180-\beta) - \mathrm{crd}(\beta)\,\mathrm{crd}(180-\alpha)$$

Which Katz claims can easily be shown to be equivalent to the well-known sine difference formula

$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

**Heron's Formula(s)**

Heron of Alexandria worked out a lot of formulas for the areas of plane figures, the most famous of which is

$$\textbf{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where $s=\frac{1}{2}(a+b+c)$ and the lengths of the three sides are $a, b$ and $c$. Some have attributed this formula to Archimedes although it appears in Heron's *Metrica*.

Heron also gave formulas for $A_n$, the areas of regular polygons with $n$ sides

$$A_3 \approx \frac{13}{30}a^2 \quad A_5 \approx \frac{5}{3}a^2 \quad A_7 \approx \frac{43}{12}a^2$$

He used $A=\frac{11}{14}d^2$ as the area of a circle of diameter $d$, making use of Archimedes approximation of $\frac{22}{7}$ for $\pi$.