History of Mathematics

Math 395 Spring 2010
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Fowler 310 MWF 10:30am - 11:25am
http://faculty.oxy.edu/ron/math/395/10/

Class 10: **Wednesday February 10**

TITLE Apollonius and his Conics
CURRENT READING: Katz, §4.4-4.5
NEXT READING: Katz, §5.1-5.3

Homework for Friday February 12
Katz, p. 91. #8, #19, #26 and #35. EXTRA CREDIT: #20.

SUMMARY
Apollonius of Perga (c. 250 – 175 BCE) along with Archimedes and Euclid forms the "holy trinity" of Greek mathematicians.

"The Great Geometer"
Appollonius coined the terms "parabola," "hyperbola" and "ellipse" in his seminal work *On Conics*.

What's amazing is the number of results that he was able to achieve without knowing about a coordinate system or algebra. It is all based on geometric reasoning.

The Cube Doubling Problem Is Solved
Recall that one of the (three) classic famous problems of antiquity is given a cube, how does one construct a cube of double the volume. Basically, this is about constructing a length which is $\sqrt[3]{2}$ of another length.

Hippocrates had showed that one needs to obtain lengths which are in the ratio $a{:}x=x{:}y=y{:}2a$ where a is your original length and x is a length so that $(a{:}x)^3=1{:}2$.

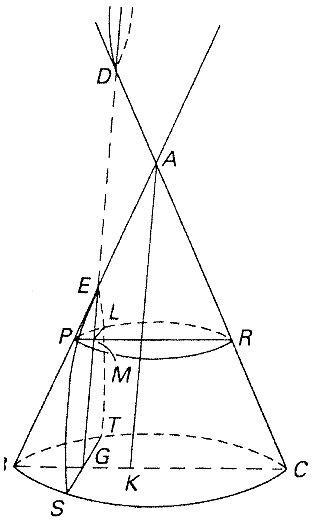
As Katz notes on page 112, algebraically, this is equivalent to solving simultaneous any two of the following equations $x^2 = ay$, $y^2 = 2ax$ or $2a^2 = xy$. Each of these curves happens to be a conic section (**name them**), so the cube doubling problem can be thought of as a curve intersection problem.

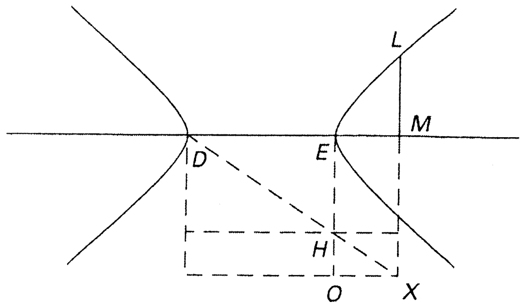
Exercise

Show that the solution of these simultaneous equations leads to $x=\sqrt[3]{2}\,a$

Apollonius Definitions Of The Conics

Hyperbola





$$\frac{DE}{EH} = \frac{AK^2}{BK \cdot KC}$$

$$\frac{AK}{BK} = \frac{EG}{BG} = \frac{EM}{MP} \quad \text{and} \quad \frac{AK}{KC} = \frac{DG}{GC} = \frac{DM}{MR}.$$

$$\frac{DE}{EH} = \frac{EM \cdot DM}{MP \cdot MR}.$$

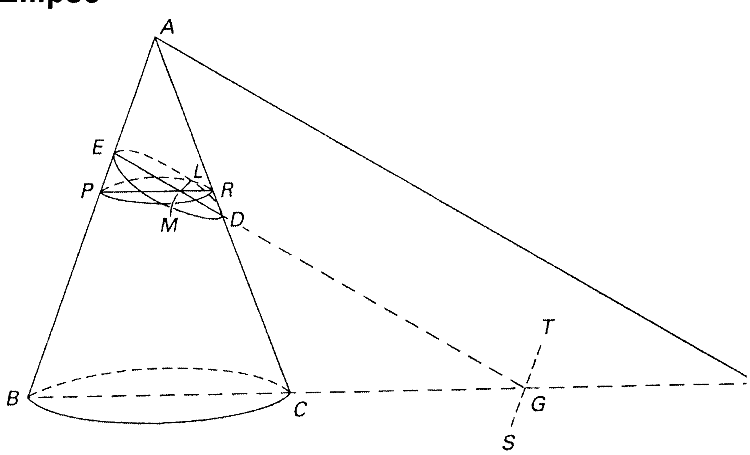
$$\frac{DE}{EH} = \frac{DM}{MX} = \frac{DM}{EO} = \frac{EM \cdot DM}{EM \cdot EO}.$$

$MP \cdot MR = EM \cdot EO$ $LM^2 = EM \cdot EO$

$EO = EH + HO$

(*yperboli* or "exceeding")

Ellipse



$$\frac{DE}{EH} = \frac{DM}{MX} = \frac{DM}{EO} = \frac{EM \cdot DM}{EM \cdot EO}$$

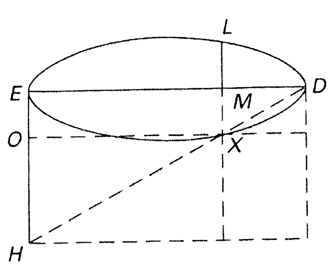
$MP \cdot MR = EM \cdot EO$

$LM^2 = EM \cdot EO$

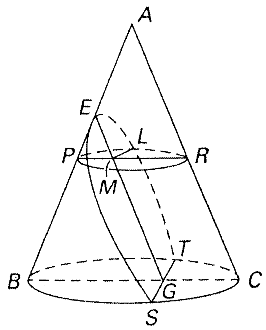
$EO = EH - HO$

(*ellipsis* or "deficient")

Let
LM=y
EM=x
EH=p
DE=2a



Parabola



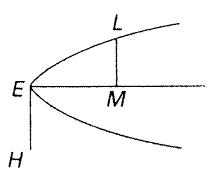
$$\frac{EH}{EA} = \frac{BC^2}{BA \cdot AC}.$$

$$\frac{EH}{EA} = \frac{MR \cdot PM}{EA \cdot EM}$$

$$\frac{EH}{EA} = \frac{EH \cdot EM}{EA \cdot EM}$$

$$MR \cdot PM = EH \cdot EM$$

$$LM^2 = EH \cdot EM$$



Let
LM=y
EM=x
EH=p

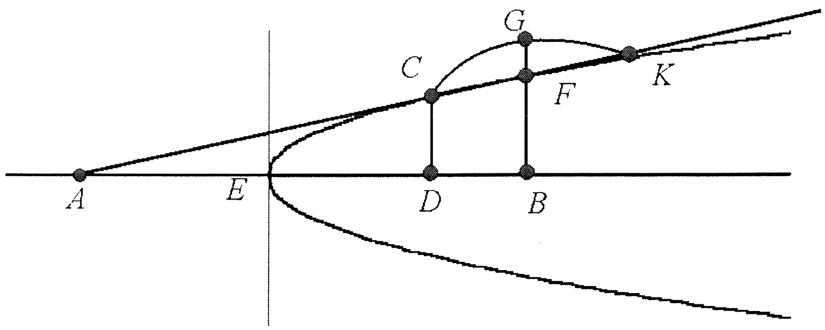
Let's derive the algebraic formulas for the ellipse, parabola and hyperbola

$$y^2 = x\left(p + \frac{p}{2a}x\right) \quad y^2 = x\left(p - \frac{p}{2a}x\right) \quad y^2 = px$$

GroupWork

Chapter 4, Problems #19-20. Let's try and prove the following results using Calculus (and modern coordinate systems)

Proposition I-33. If AC is constructed, where $|AE| = |ED|$, then AC is tangent to the parabola.



Proposition I-34. (ellipse) Choose A so that

$$\frac{|AH|}{|AG|} = \frac{|BH|}{|BG|}$$

Then AC is tangent to the ellipse at C.