History of Mathematics

Class 7: **Wednesday February 3**

TITLE Euclid's *Elements* Book II and Geometric Algebra
CURRENT READING: Katz, §3.3

Homework for Friday February 5
Katz, p. 47-48. #2, #8, #14, #15, #20.

SUMMARY
We will examine the multiple results that we recognize as algebraic results, that Euclid proved and understood using geometric principles.

Euclidean tools: compass and straight-edge
Much of strength of the achievement of Euclid is drawn from the idea that the tools he used (compass and straight-edge) were so simple that it makes his propositions easy to understand and to duplicate.

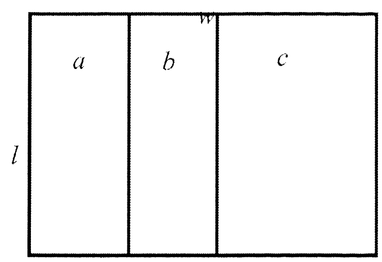
The Three Famous Problems
The duplication of the cube: to construct an edge of a cube having twice the volume of a given cube.
The trisection of an angle: to divide a given arbitrary angle into three equal parts.
The quadrature of the circle: to construct a square have an area equal to that of a given circle.

Book II of *Elements*
The book is primarily involved with what is often called "geometric algebra" and is reminiscent of the work of the Babylonians. In fact, there is debate on how much of Book II is really just a compilation of previous work from Babylonian mathematicians.

Definition. *Any rectangle is said to be* <u>*contained*</u> *by the two straight lines forming the right angle.*

The point to notice here is that Euclid does not think of the two straight lines as two lengths that are multiplied together to produce an area. There is no process or concept of an arbitrary length being multiplied by another to produce an area.

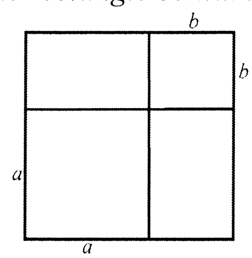
Proposition II-1. *If there are two straight lines, and one of them is cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the sum of the rectangles contained by the uncut straight line and each of the segments.*



Exercise

What algebraic property does the figure represent?

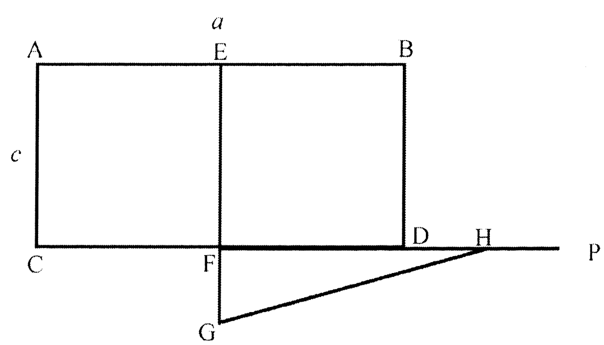
Proposition II-4. *If a straight line is cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.*



EXAMPLE

Can you see how this figure represent the binomial theorem for $n=2$?

Proposition II-14. *To construct a square equal to a given rectilinear figure.*
Algebraically, this corresponds to find a length x which solves $x^2 = ac$

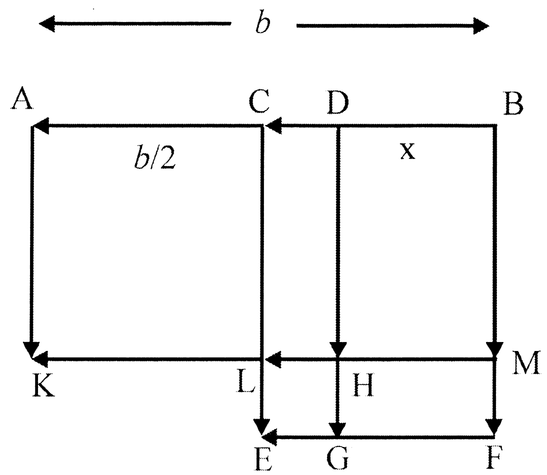


Proof. Assume $a > c$. Solve $x^2 = ac$. Construct at the midpoint of AB, and produce the line EG of length $(a + c)/2$. Therefore length of the segment FG is $(a - c)/2$. Extend the line CD to P and construct the line GH of length $(a + c)/2$ (H is on this line.). By the Pythagorean theorem the length of the line FH has square given by

$$\left(\frac{a+c}{2}\right)^2 - \left(\frac{a-c}{2}\right)^2 = ac$$

GroupWork

Prove Proposition II-5: *If a straight line is cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.*



Algebraically, this translates to: $(b-x)x + (b/2 - x)^2 = (b/2)^2$. How?