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# History of Mathematics

Math 395 Spring 2010  
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Fowler 310 MWF 10:30am - 11:25am  
<http://faculty.oxy.edu/ron/math/395/10/>

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## *Class 6: Monday February 1*

**TITLE** Introduction to Euclid and the *Elements*

**CURRENT READING:** Katz, §3.1

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### **Homework #2 for Friday February 5**

**Katz, p. 47-48. #2, #8, #12, #14, #20.**

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### **SUMMARY**

We will be introduced to Euclid (fl. 300 BCE), and the most famous mathematical book of all time: Euclid's *Elements*.

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### **Epistemology**

How do we know what we know about Euclid and the *Elements*?

What do we know about Euclid?

What do we know about the *Elements*?

### **Postulates**

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than to right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the to right angles.

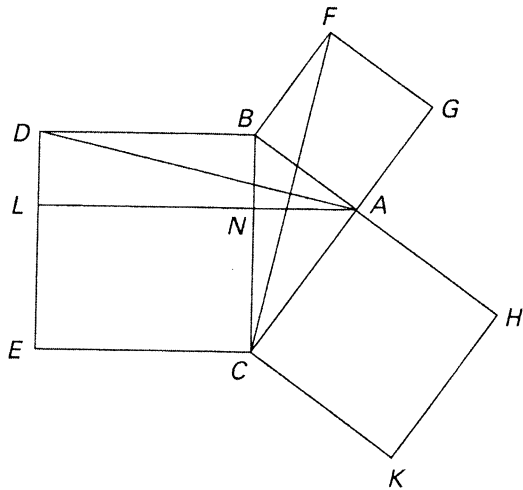
### **Axioms**

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Book I (of XIII) of the *Elements* is really all about getting to the point where Pythagoras' Theorem can be proved.

**PROPOSITION I-47**

*In right-angled triangles the square on the hypotenuse is equal to the sum of the squares on the legs.*



**PROPOSITION I-46**

*On a given straight line to describe a square.*

**PROPOSITION I-4**

*If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal sides also equal, then the two triangles are congruent.*

**PROPOSITION I-41**

*If a parallelogram has the same base with a triangle and is in the same parallels, the parallelogram is double the triangle.*

**PROPOSITION I-27**

*If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.*

*same parallels, the parallelogram is double the triangle.*

**PROPOSITION I-16**

*In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.*

**PROPOSITION I-34**

*In parallelograms the opposite sides and angles equal one another, and the diameter bisects the areas.*

**PROPOSITION I-29**

*A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.*

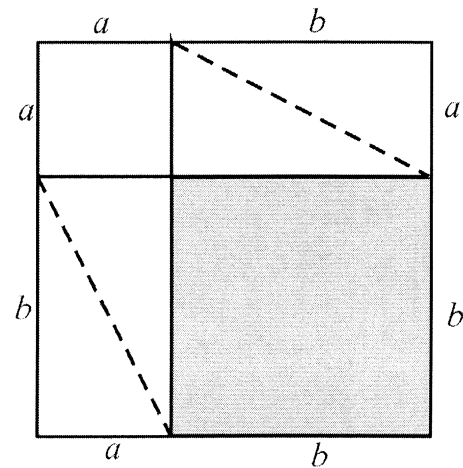
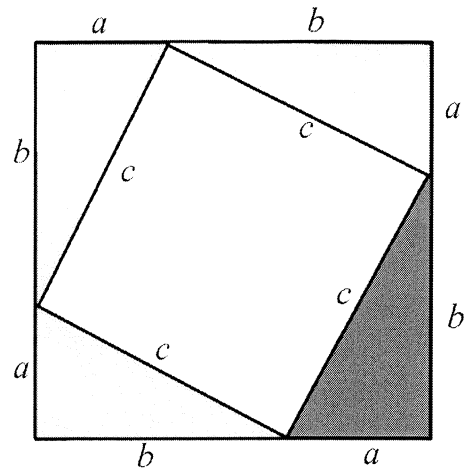
**GroupWork**

Let's repeat Katz's discussion (pp. 54-59) and work backwards through the propositions (theorems) that one needs in order to prove Pythagoras' Theorem.

**Alternate Proof(s) of Pythagoras Theorem**

G. Donald Allen claims there are over 300 published proofs of this theorem, and gives the following very pretty one

$$\begin{aligned}(a + b)^2 &= c^2 + 4\left(\frac{1}{2}ab\right) \\ a^2 + 2ab + b^2 &= c^2 + 2ab \\ a^2 + b^2 &= c^2\end{aligned}$$



This proof is based upon Books I and II of Euclid's *Elements*, and is supposed to come from the figure to the right. Euclid allows the decomposition of the square into the two boxes and two rectangles. The rectangles are cut into the four triangles shown in the figure.

Then the triangle are reassembled into the first figure.