



Addition is executed by grouping



**Exercise**

What sum is represented by the figures above?

**Multiplication and Division** are both binary

Example: Multiply:  $47 \times 24$

47	×	24	
47		1	doubling process
94		2	
188		4	
376		8	*
752		16	*

Selecting 8 and 16 (i.e.  $8 + 16 = 24$ ), we have

$$\begin{aligned}
 24 &= 16 + 8 \\
 47 \times 24 &= 47 \times (16 + 8) \\
 &= 752 + 376 \\
 &= 1128
 \end{aligned}$$

Example:  $329 \div 12$

329	÷	12	
12		1	doubling
24		2	
48		4	
96		8	
192		16	
384		32	
			-192
			137
			-96
			41
			-24
			17
			-12
			5

Now

$$\begin{aligned}
 329 &= 16 \times 12 + 8 \times 12 + 2 \times 12 + 1 \times 12 + 5 \\
 &= (16 + 8 + 2 + 1) \times 12 + 5
 \end{aligned}$$

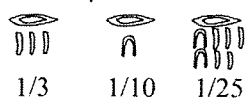
So,

$$329 \div 12 = 27 \frac{5}{12} = 27 + \frac{1}{3} + \frac{1}{12}$$

**Fractions**

Fractions were written with a symbol, called the **horus-eye**, in the numerator over the hieroglyphics for the number in the denominator. In hieratic, a dot was placed over the hieratic symbol for the number in the denominator.

Examples



There were special symbols for  $1/2$ ,  $2/3$  and  $3/4$



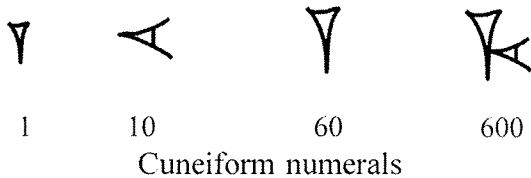
The Katz book uses  $\bar{n}$  to represent  $1/n$  and  $\bar{\bar{3}}$  to represent  $2/3$

**Babylonian Mathematics**

- . They used no zero.
- . More general fractions, though not all fractions, were admitted.
- . They could extract square roots.
- . They could solve linear systems.
- . They worked with Pythagorean triples.
- . They solved cubic equations with the help of tables.
- . They studied circular measurement.
- . Their geometry was sometimes incorrect.

For enumeration the Babylonians used symbols for 1, 10, 60, 600, 3,600, 36,000, and 216,000.

Below are four of the symbols..



They did arithmetic in base 60, **sexagesimal**

For our purposes we will use just the first two symbols

$$\nabla = 1 \qquad \nwarrow = 10$$

All numbers will be formed from these.

Example:

$$\begin{array}{l} \nwarrow \nwarrow \quad \nabla \nabla \nabla \\ \nwarrow \nwarrow \nwarrow \quad \nabla \nabla \nabla \nabla \end{array} = 57$$

Note the notation was **positional** and **sexagesimal**:

$$\nwarrow \nwarrow \quad \nwarrow \nwarrow = 20 \cdot 60 + 20 \quad \text{and} \quad \nabla \nabla \quad \nabla \nabla \quad \nwarrow \nabla = 2 \cdot 60^2 + 2 \cdot 60 + 11 = 7,331$$

**Notation**

We will write these numbers as **20,20** and **2,2,11**

**In general**

It should be noted that Babylonians did not use a zero but just left a space if a number was missing a particular power (Katz, 12).

**Why Base 60?**

Many different reasons were given for this choice, some are:

1. The number of days, 360, in a year gave rise to the subdivision of the circle into 360 degrees, and that the chord of one sixth of a circle is equal to the radius gave rise to a natural division of the circle into six equal parts. This in turn made 60 a natural unit of counting.
2. The Babylonians used a 12 hour clock, with 60 minute hours.
3. The base 60 provided a convenient way to express fractions from a variety of systems as may be needed in conversion of weights and measures.
4. The number 60 is the product of the number of planets (5 known at the time) by the number of months in the year, 12.
5. The combination of the duodecimal system (base 12) and the base 10 system leads naturally to a base 60 system.

**Babylonian Computations**

**Addition**

**Exercise**

Show that  $23,37+41,32=1,5,9$

**Multiplication**

**Exercise**

Show that  $34 \times 9 = 5,6$

**Division and Fractions**

The Babylonians only dealt with **regular** sexagesimal numbers, that is numbers whose reciprocal is a terminating sexagesimal fraction (Katz, 14).

**EXAMPLE**

$$\frac{1}{6} = \frac{10}{60} = ;\prec$$

$$\frac{1}{9} = ;\prec\prec\prec\prec\prec\prec$$

**Exercise**

How would you compute  $1/8$  as a sexagesimal fraction?

These reciprocals were often calculated with the assistance of tables.

A table of all products equal to sixty has been found.

2	30	16	3, 45
3	20	18	3,20
4	15	20	3
5	12	24	2,30
6	10	25	2,25
8	7,30	27	2,13,20
9	6,40	30	2
10	6	32	1;52,30
12	5	36	1,40
15	4	40	1,30

From the table we can see that

$$8 \times 7;30 = 8 \times \left(7 + \frac{30}{60}\right) = 60$$

It can also be used to compute reciprocals

$$\frac{1}{8} = 0;7,30 = \frac{7}{60} + \frac{30}{60^2}$$

**Exercise**

What would an equivalent table look like for our decimal system?

(HINT: You only keep fractions which have terminating decimal representations)