CHAPTER 2

1. $125 = \rho\kappa\epsilon$, $62 = \xi\beta$, $4821 = \ '\delta\omega\kappa\alpha$, $23,855 = M^{\beta}\ '\gamma\omega\nu\epsilon$
2. $\frac{8}{9} = \angle\ \acute{\gamma}\ \acute{\iota}\acute{\eta}$ $(8/9 = 1/2 + 1/3 + 1/18)$
3. The answer is in the back of the text (except the last character should be an η instead of a β). The basic idea is that $200/9 = 22\frac{2}{9} = 22 + 1/6 + 1/18$.
4. The average of a and c is $1/4+1/16+1/64$. The average of b and d is $1/2+1/4+1/8+1/16$. The product of the two averages is $1/8 + 1/16 + 1/32 + 1/64 + 1/32 + 1/64 + 1/128 + 1/256 + 1/128 + 1/256 + 1/512 + 1/1024$, or $1/4 + 59/1024$. This is slightly less than the given answer of $1/4 + 1/16$.
5. Since $AB = BC$; since the two angles at B are equal; and since the angles at A and C are both right angles, it follows by the angle-side-angle theorem that $\triangle EBC$ is congruent to $\triangle SBA$ and therefore that $SA = EC$.
6. Because both angles at E are right angles; because AE is common to the two triangles; and because the two angles CAE are equal to one another, it follows by the angle-side-angle theorem that $\triangle AET$ is congruent to $\triangle AES$. Therefore $SE = ET$.
7. The distance from the center of the pyramid to the tip of the shadow is $378 + 342 = 720$ feet. Therefore the height of the pyramid is $6/9 = 2/3$ of this value, or 480 feet.
8. $T_n = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. Therefore the oblong number $n(n + 1)$ is double the triangular number T_n.
9. $n^2 = \frac{(n-1)n}{2} + \frac{n(n+1)}{2}$, and the summands are the triangular numbers T_{n-1} and T_n.
10. $\frac{8n(n+1)}{2} + 1 = 4n^2 + 4n + 1 = (2n + 1)^2$.
11. Suppose $a^2 + b^2 = c^2$. Suppose a is odd. Then a^2 is odd. If b is odd, then b^2 is odd and c^2 is even, so c is even. If b is even, then b^2 is even and c^2 is odd, so c is odd. A similar result holds if c is odd.
12. Examples using the first formula are (3,4,5), (5,12,13), (7,24,25), (9,40,41), (11,60,61). Examples using the second formula are (8,15,17), (12,35,37), (16,63,65), (20,99,101), (24,143,145).
13. Let us assume that the second leg is commensurable to the first and let b, a be numbers representing the two legs (in terms of some unit). We may as well assume that b and a are relatively prime. Since the hypotenuse is double the first leg, we have $b^2 + a^2 = (2a)^2 = 4a^2$, or $b^2 = 3a^2$. Since b^2 is a multiple of 3, it must also be a multiple of 9, so $b^2 = 9c^2$ and $b = 3c$. Then $9c^2 = 3a^2$, or $a^2 = 3c^2$. This implies that a^2 is a multiple of 9, so that a is a multiple of 3. But then both a and b are multiples of 3, contradicting the fact that they are relatively prime.
14. Since similar segments are to their corresponding circles in the same ratio, the areas of similar segments are to one another as the squares on the diameters of the circles. Thus, the areas of similar segments are also to one another as the squares on the radii of the circles. But in similar segments, the triangles formed by the two radii and chords are similar triangles. Thus the chord of one segment is to the chord in the similar segment as the radius of the first circle to the radius of the second. That is, the squares on the

radii are to one another as the squares on the chords. Therefore, the areas of similar segments are to one another as the squares on their chords.

15. By exercise 14, the area of segment BD is the area of segment AB as the square on BD is the square on AB. But this ratio is equal to 3. Thus, the area of segment BD is three times the area of segment AB, or is equal to the sum of the areas of segments AB, AC, and CD. Therefore, the area of lune is equal to the difference between the area of the large segment and the area of segment BD. But this is equal to the difference between the area of the large segment and the areas of the three small segments, which is in turn equal to the area of the trapezoid. To construct the trapezoid, note that one can certainly construct a line segment equal to $\sqrt{3}$ times the length of a given line segment. To place this line segment both parallel to the original one and such that the lines connecting the endpoints of the two segments are each equal to the original line segment, we simply need to find the distance between the two segments. And that can be constructed by using the Pythagorean Theorem applied to the triangle whose hypotenuse is equal to the original segment and one leg of which is equal to half the difference between the new line segment and the original one. To circumscribe a circle around this trapezoid, note that one can construct a circle through three points, Say B, A, and C. By the symmetry of the trapezoid, this circle will also go through point D.

21. If one equates the times of the two runners, where d is the distance traveled by Achilles, the equation is $d/10 = (d - 500)/(1/5)$. This is equivalent to $49d = 25,000$, so $d = 510.2$ yards. Since Achilles is traveling at 10 yards per second, this will take him 51.02 seconds.