CHAPTER 1

1. The answers are given in the answer section of the text. For the Egyptian hieroglyphics, 375 is three hundreds, seven tens and five ones, while 4856 is four thousands, eight hundreds, five tens, and six ones. For Babylonian cuneiform, note that $375 = 6 \times 60 + 15$ while $4856 = 1 \times 3600 + 20 \times 60 + 56$.

2.

$$\begin{array}{cc}
1 & 34 \\
{}'2 & 68 \\
4 & 136 \\
8 & 272 \\
\underline{{}'16} & \underline{544} \\
18 & 612
\end{array}$$

$$\begin{array}{ccc}
1 & 5 & \\
10 & 50' & \text{(multiply by 10)} \\
2 & 10 & \text{(double first line)} \\
4 & 20 & \text{(double third line)} \\
8 & 40' & \text{(double fourth line)} \\
\overline{2} & 2\ \overline{2}' & \text{(halve first line)} \\
\underline{\overline{10}} & \underline{\overline{2}}' & \text{(invert third line)} \\
18\ \overline{2}\ \overline{10} & 93 &
\end{array}$$

3.

$$\begin{array}{ccc}
1 & \overline{2} & \overline{14} \\
\overline{2} & \overline{4} & \overline{28} \\
\overline{4} & \underline{\overline{8}} & \underline{\overline{56}} \\
 & 1 &
\end{array}$$

4.

$$\begin{array}{cc}
1 & \overline{28} \\
\overline{2} & \overline{56} \\
\overline{4} & \underline{\overline{112}} \\
 & \overline{16}
\end{array}$$

5. We multiply 10 by $\overline{\overline{3}}\ \overline{30}$:

1		$\overline{\overline{3}}$	$\overline{30}$	
′2	1	$\overline{3}$	$\overline{15}$	
4	2	$\overline{\overline{3}}$	$\overline{10}$	$\overline{30}$
′8	5	$\overline{2}$	$\overline{10}$	

The total of the two marked lines is then 7, as desired.

6.

1	7	$\overline{2}$	$\overline{4}$	$\overline{8}$
2	15	$\overline{2}$	$\overline{4}$	
4	31	$\overline{2}$		
8	63			
$\overline{\overline{3}}$	5	$\overline{4}$		

Note that the sum of the three last terms in the second column is $99\ \overline{2}\ \overline{4}$. We therefore need to figure out by what to multiply $7\ \overline{2}\ \overline{4}\ \overline{8}$ to give $\overline{4}$ so that we get a total of 100. But since we know from the fourth line that multiplying that value by 8 gives 63, we also know that multiplying it by $\overline{63}$ gives $\overline{8}$. Thus the required number is double $\overline{63}$, which is $\overline{42}\ \overline{126}$. Thus the final result of our division is $12\ \overline{\overline{3}}\ \overline{42}\ \overline{126}$.

7.

1	$7\ \overline{2}\ \overline{4}\ \overline{8}$
2	$15\ \overline{2}\ \overline{4}$
′4	$31\ \overline{2}$
′8	63
$\underline{'\overline{\overline{3}}}$	$\underline{4\ \overline{\overline{3}}\ \overline{3}\ \overline{6}\ \overline{12}}$
$12\ \overline{\overline{3}}$	$98\ \overline{2}\ \overline{\overline{3}}\ \overline{3}\ \overline{6}\ \overline{12}$ $99\ \overline{2}\ \overline{4}$

8.

$2 \div 11$	1	11	$2 \div 23$	1	23
	$\overline{\overline{3}}$	$7\ \overline{3}$		$\overline{\overline{3}}$	$15\ \overline{3}$
	$\overline{3}$	$3\ \overline{\overline{3}}$		$\overline{3}$	$7\ \overline{\overline{3}}$
	$\overline{6}$	$1\ \overline{\overline{3}}\ \overline{6}'$		$\overline{6}$	$3\ \overline{2}\ \overline{3}$
	$\overline{66}$	$\overline{6}'$		$\overline{12}$	$1\ \overline{2}\ \overline{4}\ \overline{6}'$
				$\overline{276}$	$\overline{12}'$
	$\overline{6}\ \overline{66}$	2		$\overline{12}\ \overline{276}$	2

9. $x + \frac{1}{7}x = 19$. Choose $x = 7$; then $7 + \frac{1}{7} \cdot 7 = 8$. Since $19 \div 8 = 2\frac{3}{8}$, the correct answer is $2\frac{3}{8} \times 7 = 16\frac{5}{8}$.
10. $(x + \frac{2}{3}x) - \frac{1}{3}(x + \frac{2}{3}x) = 10$. In this case, the "obvious" choice for x is $x = 9$. Then 9 added to 2/3 of itself is 15, while 1/3 of 15 is 5. When you subtract 5 from 15, you get 10. So in this case our "guess" is correct.
11. The equation here is $(1 + \frac{1}{3} + \frac{1}{4})x = 2$. Therefore. we can find the solution by dividing 2 by $1 + \frac{1}{3} + \frac{1}{4}$. We set up that problem:

1	$1\ \overline{2}\ \overline{4}$
$\overline{\overline{3}}$	$1\ \overline{18}$
$\overline{3}$	$\overline{2}\ \overline{36}$
$\overline{6}$	$\overline{4}\ \overline{72}$
$\overline{12}$	$\overline{8}\ \overline{144}$

The sum of the numbers in the right hand column beneath the initial line is $1\frac{141}{144}$. So we need to find multipliers giving us $\frac{3}{144} = \overline{144}\ \overline{72}$. But $1\ \overline{3}\ \overline{4}$ times 144 is 228. It follows that multiplying $1\ \overline{3}\ \overline{4}$ by $\overline{228}$ gives $\overline{144}$ and multiplying by $\overline{114}$ gives $\overline{72}$. Thus, the answer is $1\ \overline{6}\ \overline{12}\ \overline{114}\ \overline{228}$.

12. The equation is $x + 2x = 9$. This reduces to $3x = 9$, so the answer is 3.
13. Since x must satisfy $100 : 10 = x : 45$, we would get that $x = \frac{45 \times 100}{10}$; the scribe breaks this up into a sum of two parts, $\frac{35 \times 100}{10}$ and $\frac{10 \times 100}{10}$.
14. The ratio of the cross section area of a log of 5 handbreadths in diameter to one of 4 handbreadths diameter is $5^2 : 4^2 = 25 : 16 = 1\frac{9}{16}$. Thus, 100 logs of 5 handbreadths diameter are equivalent to $1\frac{9}{16} \times 100 = 156\frac{1}{4}$ logs of 4 handbreadths diameter.
16. The modern formula for the surface area of a half-cylinder of diameter d and height h is $A = \frac{1}{2}\pi dh$. Similarly, the modern formula for the surface area of a hemisphere of diameter d is $A = \frac{1}{2}\pi d^2$. These formulas are identical if $h = d$.
17. $7/5 = 1;24$ $\quad 13/15 = 0;52$ $\quad 11/24 = 0;27,30$ $\quad 33/50 = 0;39,36$

18. $0;22,30 = 3/8 \quad 0;08,06 = 27/200 \quad 0;04,10 = 5/72 \quad 0;05,33,20 = 5/54$
19. Since $3 \times 18 = 54$, which is 6 less than 60, it follows that the reciprocal of 18 is $3\frac{1}{3}$, or, putting this in sexagesimal notation, 3,20. Since 60 is $(1\frac{7}{8}) \times 32$, and $\frac{7}{8}$ can be expressed as 52,30, the reciprocal of 32 is 1,52,30. Since $60 = 1\frac{1}{9} \times 54$, and $\frac{1}{9}$ can be expressed as $\frac{1}{10}+\frac{1}{90} = \frac{6}{60}+\frac{40}{3600} = 0;06,40$, the reciprocal of 54 is $1,06,40$. Also, because $60 = \frac{15}{16}\times 64$, the reciprocal of 64 is $\frac{15}{16}$. Since $\frac{1}{16} = 3,45$, we get that $\frac{15}{16} = 56,15$. If the only prime divisors of n are 2, 3, 5, then n is a regular sexagesimal.
20. $25 \times 1,04 = 1,40 + 25,00 = 26,40$. $18 \times 1,21 = 6,18 + 18,00 = 24,18$. $50 \div 18 = 50 \times 0;3,20 = 2;30 + 0;16,40 = 2;46,40$. $1,21 \div 32 = 1,21 \times 0;01,52,30 = 1;21 + 1;10,12 + 0;00,40,30 = 2;31,52,30$.
21. Since the length of the circumference C is given by $C = 4a$, and because $C = 6r$, it follows that $r = \frac{2}{3}a$. The length T of the long transversal is then $T = r\sqrt{2} = (\frac{2}{3}a)(\frac{17}{12}) = \frac{17}{18}a$. The length t of the short transversal is $t = 2(r - \frac{t}{2}) = 2a(\frac{2}{3} - \frac{17}{36}) = \frac{7}{18}a$. The area A of the barge is twice the difference between the area of a quarter circle and the area of the right triangle formed by the long transversal and two perpendicular radii drawn from the two ends of that line. Thus
$$A = 2\left(\frac{C^2}{48} - \frac{r^2}{2}\right) = 2\left(\frac{a^2}{3} - \frac{2a^2}{9}\right) = \frac{2}{9}a^2.$$
22. Since the length of the circumference C is given by $C = 3a$, and because $C = 6r$, it follows that $r = \frac{a}{2}$. The length T of the long transversal is then $T = r\sqrt{3} = (\frac{a}{2})(\frac{7}{4}) = \frac{7}{8}a$. The length t of the short transversal is twice the distance from the midpoint of the arc to the center of the long transversal. If we set up our circle so that it is centered on the origin, the midpoint of the arc has coordinates $(\frac{r}{2}, \frac{\sqrt{3}r}{2})$ while the midpoint of the long transversal has coordinates $(\frac{r}{4}, \frac{\sqrt{3}r}{4})$. Thus the length of half of the short transversal is $\frac{r}{2}$ and then $t = r = \frac{a}{2}$. The area A of the bull's eye is twice the difference between the area of a third of a circle and the area of the triangle formed by the long transversal and radii drawn from the two ends of that line. Thus
$$A = 2\left(\frac{C^2}{36} - \frac{1}{2}\frac{r}{2}T\right) = 2\left(\frac{9a^2}{36} - \frac{1}{2}\frac{a}{4}\frac{7a}{8}\right) = 2a^2\left(\frac{1}{4} - \frac{7}{64}\right) = \frac{9}{32}a^2.$$
23. If a is the length of one of the quarter-circle arcs defining the concave square, then the diagonal is equal to the diameter of that circle. Since the circumference is equal to $4a$, the diameter is one-third of that circumference, or $1\frac{1}{3}a$. The transversal is equal to the diagonal of the circumscribing square less the diameter of the circle (which is equal to the side of the square). Since the diagonal of a square is approximated by 17/12 of the side, the transversal is therefore equal to 5/12 of the diameter, or $\frac{5}{12}\frac{4}{3}a = \frac{5}{9}a$.
24. $1;24,51,10 = 1+\frac{24}{60}+\frac{51}{3600}+\frac{10}{216000} = 1+0.4+0.0141666666+0.0000462962 = 1.414212963$. On the other hand, $\sqrt{2} = 1.414213562$. Thus the Babylonian value differs from the true value by approximately 0.00004%.
25. $\sqrt{3} = \sqrt{2^2-1} \approx 2 - \frac{1}{2}\cdot 1 \cdot \frac{1}{2} = 2 - 0;15 = 1;45$. Since an approximate reciprocal of 1;45 is 0;34,17.09, we get further that $\sqrt{3} = \sqrt{(1;45)^2 - 0;03,45} = 1;45 -$

$(0;30)(0;03,45)(0;34,17.09) = 1;45 - 0;01,04,17,09 = 1;43,55,42,51$, which we truncate to 1;43,55,42 because we know this value is a slight over-approximation.

26. $v + u = 1;48 = 1\frac{4}{5}$ and $v - u = 0;33,20 = \frac{5}{9}$. So $2v = 2;21,20$ and $v = 1;10,40 = \frac{106}{90}$. Similarly, $2u = 1;14,40$ and $u = 0;37,20 = \frac{56}{90}$. Multiplying by 90 gives $x = 56$, $d = 106$. In the second part, $v + u = 2;05 = 2\frac{1}{12}$ and $v - u = 0;28,48 = \frac{12}{25}$. So $2v = 2;33,48$ and $v = 1;16,54 = \frac{769}{600}$. Similarly, $2u = 1;36,12$ and $u = 0;48,06 = \frac{481}{600}$. Multiplying by 600 gives $x = 481$, $d = 769$. Next, if $v = \frac{481}{360}$ and $u = \frac{319}{360}$, then $v + u = 2\frac{2}{9} = 2;13,20$. Finally, if $v = \frac{289}{240}$ and $u = \frac{161}{240}$, then $v + u = 1\frac{7}{8} = 1;52,30$.

27. The equations for u and v can be solved to give $v = 1;22,08,27 = \frac{295707}{216000} = \frac{98569}{72000}$ and $u = 0;56,05,57 = \frac{201957}{216000} = \frac{67319}{72000}$. Thus the associated Pythagorean triple is 67319, 72000, 98569.

28. The two equations are $x^2 + y^2 = 1525$; $y = \frac{2}{3}x + 5$. If we substitute the second equation into the first and simplify, we get $13x^2 + 60x = 13500$. The solution is then $x = 30$, $y = 25$.

29. If we guess that the length of the rectangle is 60, then the width is 45 and the diagonal is $\sqrt{60^2 + 45^2} = 75$. Since this value is $1\frac{7}{8}$ times the given value of 40, the correct length of the rectangle should be $60 \div 1\frac{7}{8} = 32$. Then the width is 24.

30. One way to solve this is to let x and $x - 600$ be the areas of the two fields. Then the equation is $\frac{2}{3}x + \frac{1}{2}(x - 600) = 1100$. This reduces to $\frac{7}{6}x = 1400$, so $x = 1200$. The second field then has area 600.

31. Let x be the weight of the stone. The equation to solve is then $x - \frac{1}{7}x - \frac{1}{13}(x - \frac{1}{7}x) = 60$. We do this using false position twice. First, set $y = x - \frac{1}{7}x$. The equation in y is then $y - \frac{1}{13}y = 60$. We guess $y = 13$. Since $13 - \frac{1}{13}13 = 12$, instead of 60, we multiply our guess by 5 to get $y = 65$. We then solve $x - \frac{1}{7}x = 65$. Here we guess $x = 7$ and calculate the value of the left side as 6. To get 65, we need to multiply our guess by $\frac{65}{6} = 10\frac{1}{6}$. So our answer is $x = 7 \times \frac{65}{6} = 75\frac{5}{6}$ *gin*, or 1 *mina* $15\frac{5}{6}$ *gin*.

32. We do this in three steps, each using false position. First, set $z = x - \frac{1}{7}x + \frac{1}{11}(x - \frac{1}{7}x)$. The equation for z is then $z - \frac{1}{13}z = 60$. We guess 13 for z and calculate the value of the left side to be 12, instead of 60. Thus we must multiply our original guess by 5 and put $z = 65$. Then set $y = x - \frac{1}{7}x$. The equation for y is $y + \frac{1}{11}y = 65$. If we now guess $y = 11$, the result on the left side is 12, instead of 65. So we must multiply our guess by $\frac{65}{12}$ to get $y = \frac{715}{12} = 59\frac{7}{12}$. We now solve $x - \frac{1}{7}x = 59\frac{7}{12}$. If we guess $x = 7$, the left side becomes 6 instead of $59\frac{7}{12}$. So to get the correct value, we must multiply 7 by $\frac{715}{12}/6 = \frac{715}{72}$. Therefore, $x = 7 \times \frac{715}{72} = \frac{5005}{72} = 69\frac{37}{72}$ *gin* = 1 *mina* $9\frac{37}{72}$ *gin*.

33. Start with a square of side x and cut off a strip of width a from the right side. The remaining rectangle then has area $x^2 - ax$, or b. This rectangle can then be thought of as a square of side $x - a/2$ that is missing a small square of side $a/2$. If one adds back that small square, then the square of side $x - a/2$ has area $b + (a/2)^2$, so we can find x.

34. The equation $x - \frac{60}{x} = 7$ is equivalent to $x^2 - 60 = 7x$ or to $x^2 - 7x = 60$. The solution is then $x = \sqrt{(\frac{7}{2})^2 + 60} + \frac{7}{2} = \frac{17}{2} + \frac{7}{2} = 12$. Thus the two numbers are 12 and 5.

35. Given the appropriate coefficients, the equation becomes $\frac{4}{9}a^2 + a + \frac{4}{3}a = \frac{23}{18}$, where a

is the length of the arc. If we scale up by $\frac{4}{9}$, we get the equation $(\frac{4}{9}a)^2 + \frac{7}{3}(\frac{4}{9}a) = \frac{46}{81}$. The algorithm for this type of equation gives $\frac{4}{9}a = \sqrt{(\frac{7}{6})^2 + \frac{46}{81}} - \frac{7}{6} = \frac{25}{18} - \frac{7}{6} = \frac{2}{9}$. Thus $a = \frac{1}{2}$.

36. The equation is $\frac{2}{3}x^2 + \frac{1}{3}x = \frac{1}{3}$. To solve, we scale by $\frac{2}{3}$: $(\frac{2}{3}x)^2 + \frac{1}{3}(\frac{2}{3}x) = \frac{2}{9}$. The solution is $\frac{2}{3}x = \sqrt{(\frac{1}{6})^2 + \frac{2}{9}} - \frac{1}{6} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$. Thus $x = \frac{1}{2}$.

37. All the triangles in this diagram are similar to one another, and therefore their sides are all in the ratio 3:4:5. Therefore, $AD = \frac{4}{5}AB = 0;48 \times 0;45 = 0;36$ and $BD = \frac{3}{5}AB = 0;36 \times 0;45 = 0;27$. Similarly, $DE = \frac{4}{5}AD = 0;48 \times 0;36 = 0;28,48$. Also, $EF = \frac{4}{5}DE = 0;48 \times 0;28,48 = 0;23,02,24$. Then $DF = \frac{3}{5}DE = 0;36 \times 0;28,48 = 0;17,16,48$ and $FC = BC - BD - DF = 1;15 - 0;27 - 0;17,16,48 = 0;30,43,12$.

38. If the circumference is 60, then the radius is 10. Thus, if the distance of the chord from the circumference is x, then we have a right triangle of sides 6 and $10 - x$, with hypotenuse 10. The Pythagorean theorem leads to the equation $6^2 + (10 - x)^2 = 100$, or $x^2 + 36 = 20x$, for which the only valid solution is $x = 2$.

39. The two equations are $\ell + w = 7$, $\ell w + \frac{1}{2}\ell + \frac{1}{3}w = 15$. To put this into a standard Babylonian form, we can rewrite the second equation in the form $(\ell + \frac{1}{3})(w + \frac{1}{2}) = 15\frac{1}{6}$. We can then rewrite the first equation as $(\ell + \frac{1}{3}) + (w + \frac{1}{2}) = 7\frac{5}{6}$. Then the Babylonian algorithm yields $\ell + \frac{1}{3} = \frac{47}{12} + \sqrt{(\frac{47}{12})^2 - \frac{91}{6}} = \frac{47}{12} + \frac{5}{12} = \frac{13}{3}$. Therefore, $\ell = 4$ and so $w = 3$.