Applied Mathematics

Math 395 Spring 2009 | Fowler 301 Tue 3:00pm - 4:25pm

 | http://faculty.oxy.edu/ron/math/395/

Class 1: **Tuesday January 20**

TITLE Dimenionsl Analysis
CURRENT READING Logan, Section 1.1.1

SUMMARY
This week we will be introduced to the first thing to think about when solving a problem in applied mathematics: what are the units (and how can I get rid of them?)

Dimensional Analysis
The analysis of the dimensions of the variables and parameters in an equation.

The Pi Theorem
"Given a physical law that gives a relation among a certain number of *dimensioned* quantities, then there is an equivalent law that can be expressed as a relation among certain **dimensionless** quantities." (Logan 5)

The basic idea is that a physical law written as $f(x_1, x_2, x_3, \ldots, x_n) = 0$ can be written as $F(\pi_1, \pi_2, \ldots, \pi_m) = 0$ where x_i are dimensioned quantities (i.e. have units) while the π_i are combinations of the dimensioned quantities in such a way that the π_i are dimensionless quantiities.

What are the basic dimensioned quantities?
Generally, we can take any physical dimensioned quantity and write it as some combination of mass, length, time. These are usually denoted M, L and T and have the units kg,m and sec respectively.

EXAMPLE
Consider Taylor's law that relates the energy E released in an atomic explosion that depends on time t, the radius of the fireball r and density ρ, $g(t, r, \rho, E) = 0$

We'll use dimensional analysis to show that

$$f\left(\frac{r^5 \rho}{t^2 E}\right) = 0$$

is a dimensionless version of Taylor's law.

$\boxed{\textbf{Exercise}}$

$F = f(\rho, A, v)$ where A is cross-sectional area, v is speed and ρ is density. The force F of air resistance is related to these quantities in some way. Can you determine what it is? Use dimensional analysis!