Numerical Analysis

Math 370 Fall 1998 — MWF 11:30am - 12:25pm

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Class 11: **Monday September 28**

SUMMARY Introduction to Root Finding
READING Burden & Faires, 47–54

Example
Consider a ball constructed of wood which has a density of $\rho = 0.638$ grams per cubic cm and the radius is $r = 10$ cm. How much of the ball will be submerged when it is in water (with unit density)?

M_w = Mass of water displaced = $\int_0^d \pi(r^2 - (x-r)^2)\,dx$

M_b = Mass of ball = $4\pi r^3 \rho/3$

What's the equation which must be solved to find d, the distance below the surface the ball will float? (Produce an equation for d of the form $f(d) = 0$ with d being the only letter present.)

Question
How would you solve this equation for d?

Root-Finding

We will be looking at algorithms for the solution of equations of one variable, i.e. equations of the form $f(x) = 0$. This is often referred to as finding the **roots** of the equation $f(x) = 0$ or finding the **zeroes** of the function $f(x)$.

Bracketing The Root

How do we know where the roots of a function $f(x)$ are? How can we "bracket" a zero of $f(x)$?

GROUPWORK

The MATLAB function **brackplo** will do this for us. Go to the computers and run `brackplo` on the function you need to find zeroes of to find d. I have made a function called **sphere.m** which you can use to help you. What do you see? How many roots are there? What range did you ask `brackplo` to search on?

The Bisection Method of Bolzano

The bisection algorithm produces a sequence of approximations $\{p_n\}$ to the zero of the function $f(x)$

where $p_n = a_n + \dfrac{b_n - a_n}{2} = \dfrac{a_n + b_n}{2}$ and the n-th bracket is described by $[a_n, b_n]$

Write down the Bisection Algorithm in pseudocode here:

bisect.m

In the NMM Toolbox, we have an implementation of the bisection algorithm in **bisect.m**. Use MATLAB to find the value of d which we have been looking for which tells us how much of the pine sphere is submerged.

$d =$