Numerical Analysis

Math 370 Fall 1998 MWF 11:30am - 12:25pm
 Fowler 127

Class 8: **Monday September 21**

SUMMARY Perturbing Functions and Introduction to Matlab
CURRENT READING Recktenwald, Chapter 2

Perturbation of a Function
Suppose we have a complicated function $f(x)$ and we want know its behavior near a point a (for simplicity sake, we will let $a = 0$). We will measure how "near" a we are by using the parameter h, which we take to be very small. In some cases we may even take the limit as $h \to 0$.
If $f(x)$ is continuous, then $\lim_{h\to 0} f(x) =$ _____.
Then, we can write the behavior of $f(x)$ near $a = 0$ in terms of h using a **Maclaurin Series**

$$f(0+h) = \sum_{k=0}^{\infty} f^{(k)}(0)\frac{h^k}{k!}$$

which can be truncated as

$$f(0+h) \approx f(0) + f'(0)h + f''(0)\frac{h^2}{2}$$

Interestingly, we can write an exact expression for the truncated form

$$f(0+h) = f(0) + f'(0)h + f''(0)\frac{h^2}{2} + \mathcal{O}(h^3)$$

Yes, this last term is the same "big oh" that we have been discussing in regards to rate of convergence of a function to its limit.
We therefore have a **third** way of computing rate of convergence of a function to a limit.
Example
Show that you can use a truncated Maclaurin Expansion to prove that $\cos(h)+\frac{h^2}{2} = 1+\mathcal{O}(h^4)$

The Three Ways Of Computing Order of Convergence of a Function are

1. **Limit Method**
2. **Bounding/Inequality Method**
3. **Truncated Taylor/Maclaurin Expansion**

(next to each of the methods above, write a short note to yourself explaining the method.)
GROUPWORK
Find the limit and express the order of convergence in terms of $f(h) = c + \mathcal{O}(h^\alpha) = c + o(h^\beta)$ for $f(h) = \dfrac{1 + h - e^h}{h^2}$

Introduction to MATLAB

MATLAB is an interactive numerical computing environment. It allows both command-line instructions, and programs, which are placed in files ending with `.m`.

Today we will try running MATLAB and executing some basic operations. We will be using MATLAB as the environment in which we execute our numerical algorithms in this course.

Next class I will give you more details on MATLAB from Gerry Recktenwald's textbook. Bring a computer disk with you.