Numerical Analysis

Math 370 Fall 1998 MWF 11:30am - 12:25pm

 Fowler 127

Class 4: **Monday September 9**

SUMMARY Examples of Round-Off Errors
CURRENT READING Burden & Faires Sections 1.2 and 1.4

Loss of Significance

Consider the numbers $a = 0.54617$ and $b = 0.54601$. Compute $c = a \ominus b$ using 4-digit chopping and then 4-digit rounding arithmetic.

Compute the relative error in c using chopping and rounding arithmetic

Example
Consider two numbers x and y which have k-digit decimal representations where p digits ($p < k$) are the same. Let's write their decimal number representation $fl(x)$ and $fl(y)$ below Then let's write down the representation of $fl(fl(x) - fl(y)$

Round-off Errors in the Quadratic Formula

Recall that the common formula for the roots of a quadratic equation $ax^2 + bx + c = 0$ is

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \text{and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Round-off error can wreak havoc with the numerical implementation of this formula. Consider

$$x^2 + 62.10x + 1 = 0$$

which has the approximate roots $x_1 = -0.01610723$ and $x_2 = -62.08390$
Because of the size of the parameters in the quadratic equation, b^2 is much bigger than $4ac$, so $\sqrt{b^2 - 4ac}$ is very close to b

$b =$ $\qquad$ $b^2 =$ $\qquad$ $4ac =$ $\qquad$ $b^2 - 4ac =$

Groupwork

Using 4-digit rounding arithmetic compute the first root x_1

What's the relative error in this calculation? Solution: change the formula for x_1 so that we

don't have to subtract b from $\sqrt{b^2 - 4ac}$

Now, a new formula for $x_1 =$

Use a similar new formula to compute x_2 (using 4-digit precision) and compute the relative error in x_2

What's the problem?

Solution: Use the new formula for x_1 when you have to subtract numbers which are similar in size, use the traditional formula for the other root.

The Ultimate Quadratic Formula

$$q \equiv -\frac{1}{2}\left[b + \mathrm{sign}(b)\sqrt{b^2 - 4ac}\right]$$

where

$$\mathrm{sign(b)} = \begin{cases} 1 & b \geq 0 \\ -1 & b < 0 \end{cases}$$

and

$$x_1 = \frac{q}{a} \qquad \text{and } x_2 = \frac{c}{q}$$

Machine Precision

There is a number ϵ_m such that $1 + \delta = 1$ whenever $\delta < \epsilon_m$

What is ϵ_m equal to in **exact arithmetic**?

How would you compute the machine precision of **your** calculator? Developan algorithm.

ANNOUNCEMENTS

Class is cancelled on Friday September 11.

Quiz 2 is due on Monday September 14 in class.