Summary  Examples of Round-Off Errors
Current Reading  Burden & Faires Sections 1.2 and 1.4

Loss of Significance
Consider the numbers $a = 0.54617$ and $b = 0.54601$. Compute $c = a \oplus b$ using 4-digit chopping and then 4-digit rounding arithmetic.

Compute the relative error in $c$ using chopping and rounding arithmetic.

Example
Consider two numbers $x$ and $y$ which have $k$-digit decimal representations where $p$ digits ($p < k$) are the same. Let’s write their decimal number representation $fl(x)$ and $fl(y)$ below. Then let’s write down the representation of $fl(fl(x) - fl(y))$.

Round-off Errors in the Quadratic Formula
Recall that the common formula for the roots of a quadratic equation $ax^2 + bx + c = 0$ is

$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Round-off error can wreak havoc with the numerical implementation of this formula. Consider

$x^2 + 62.10x + 1 = 0$

which has the approximate roots $x_1 = -0.01610723$ and $x_2 = -62.08390$

Because of the size of the parameters in the quadratic equation, $b^2$ is much bigger than $4ac$, so $\sqrt{b^2 - 4ac}$ is very close to $b$.

\[
b = \quad b^2 = \quad 4ac = \quad b^2 - 4ac =
\]
**Groupwork**

Using 4-digit rounding arithmetic compute the first root $x_1$

What’s the relative error in this calculation? Solution: change the formula for $x_1$ so that we don’t have to subtract $b$ from $\sqrt{b^2 - 4ac}$

Now, a new formula for $x_1 =$

Use a similar new formula to compute $x_2$ (using 4-digit precision) and compute the relative error in $x_2$

What’s the problem? Solution: Use the new formula for $x_1$ when you have to subtract numbers which are similar in size, use the traditional formula for the other root.

**The Ultimate Quadratic Formula**

$$ q \equiv -\frac{1}{2} \left[ b + \text{sign}(b) \sqrt{b^2 - 4ac} \right] $$

where

$$ \text{sign}(b) = \begin{cases} 1 & b \geq 0 \\ -1 & b < 0 \end{cases} $$

and

$$ x_1 = \frac{q}{a} \quad \text{and} \quad x_2 = \frac{c}{q} $$

**Machine Precision**

There is a number $\epsilon_m$ such that $1 + \delta = 1$ whenever $\delta < \epsilon_m$

What is $\epsilon_m$ equal to in exact arithmetic?

How would you compute the machine precision of your calculator? Develop an algorithm.

**ANNOUNCEMENTS**

Class is cancelled on Friday September 11.

Quiz 2 is due on Monday September 14 in class.