Math 370 Fall 98

Quiz 5 — Numerical Analysis

Name: ______________________________

Date: ____________________ **Friday October 16**
Time Begun: ____________________ Ron Buckmire
Time Ended: ____________________

Topic : Analyzing Iterative Methods

The idea behind this quiz is for you to illustrate your understanding of rates of convergence of iterative methods by considering Halley's Method and Lazy Newton's Method.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. **You must work alone.**
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday October 19**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 370 Fall 1998 **EXPLAIN YOUR ANSWERS** Quiz **5**

1. *(2 points)* Recall that Lazy Newton's Method is $x_n = g(x_{n-1}) = x_{n-1} - \dfrac{f(x_n)}{f'(x_0)}$. Thinking of this iteration scheme as a Picard Iteration show that Lazy Newton's Method is a **linearly convergent**.

2. Consider **Halley's Method**, one of the very rare cubically convergent methods known. It's iterative scheme is

$$x_n = H(x_n) = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} - \frac{f''(x_{n-1})}{2f'(x_{n-1})}\left[\frac{f(x_{n-1})}{f'(x_{n-1})}\right]^2$$

a. *(2 points)* Use Halley's Method to find the root of $f(x) = x^3 - 4$ to 2 decimal places.

b. *(6 points)* Show that Halley's Method is cubically convergent by using the similar analysis discussed in class. [Remember, $H(p) = p \leftrightarrow f(p) = 0$. This will **greatly** simplify evaluating the derivatives you need to take.]