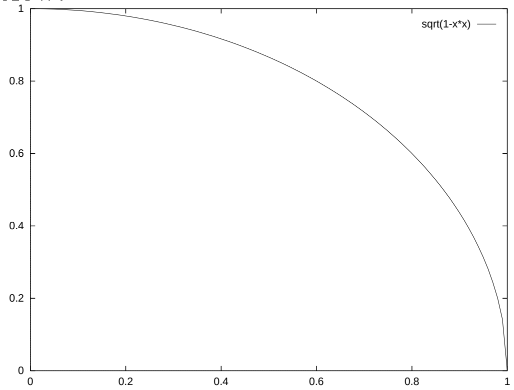
Numerical Analysis

Math 370 — MWF 2:30-3:25
Spring 1997 — 201 Fowler

(04.02.97) Numerical Integration

Warm Up

Let's try and approximate the area of a unit circle using these methods of numerical quadrature you already know.
We will do this by estimating the area under the curve $f(x) = \sqrt{1-x^2}$ from 0 to 1, depicted below:



How close did you get to the exact value of the area of the circle?
What order do you think the quadrature methods you used are? i.e. $O(h)$? $O(h^2)$?

Something else?

2-point Quadrature Rule

Let's assume that we only have 2 data points $(x_0, f(x_0))$ and $(x_1, f(x_1))$. We can use a first degree Lagrange polynomial to interpolate $f(x)$, integrate it and thus derive a quadrature rule we call ____________________________

Error Term for Trapezoid Rule

Simpson's Rule

A similar method can be used to derive the formula for Simpson's Rule. Assume that we have 3 data points (x_0, f_0), (x_1, f_1) and (x_2, f_2). Write down the formula one would have to evaluate to derive the **3-point quadrature rule** more commonly known as Simpson's Rule:

Do we believe this will get us the answer? What's wrong with it?

Let's try something else...
First, find the 4th degree Taylor expansion of $f(x)$ about the point x_1 and write it down here:

$$\int_{x_0}^{x_2} f(x)\,dx$$

Definition of Precision

A numerical quadrature method is said to have **precision** (or accuracy) n (where n is a positive integer) if $E(P_k) = 0$ for all polynomials $P_k(x)$ of degree $k \leq n$ but $E(P_{n+1}) \neq 0$

QUESTION: What degree of precision does Simpson's Rule have?

QUESTION: What degree of precision does the Trapezoidal Rule have?