Numerical Analysis

Math 370 — MWF 2:30-3:25
Spring 1997 — 127 Fowler

(03.05.97) Piecwise Polynomial Interpolation: SPLINES

Piecewise Linear Interpolation

Instead of using one interpolating polynomial of high degree, there is also the idea of using a lot of polynomials of small degree to do interpolation. For example, with the example of producing cheap and dirty sines, we said that we would use a number of linear interpolations between consecutive nodes. We say that these are **piecewise polynomials** because the "pieces" (the individual linear interpolating polynomials) are tied together at the "knots" (also known as the nodes). These pieces are also known as **splines**.

Figure

Here is a figure of piecewise linear interpolation

Given x_k, y_k for $k = 0ton$ come up with the equation for a typical piecwise linear interpolant $s_k(x)$

What is the relationship between $s_k(x)$ on the kth interval and s_{k+1} on the $(k+1)$th interval?

What are some drawbacks of doing piecewise linear interpolation instead of doing piecewise polynomial interpolation with a higher degree polynomial?

The most common piecewise polynomial interplation is done with polynomials of degree, thus leading to the famous **cubic spline** problem.

Cubic Splines
Suppose we want to have piecewise interpolating polynomials which are continuous and smooth at all the nodes. What kinds of conditions on the $s_k(x)$ and $s_{k+1}(x)$ would this entail?

Figure
Here is a figure illustrating what piecwise cubic interpolation looks like:

Given that $s_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3$ and that we have $N + 1$ data points (x_k, y_k) how many cubic splines will we use to interpolate the entire interval from x_0 to x_N? How many unknown constants do we have to compute (i.e find) in order to define all of these $s_k(x)$?

How many equations do we have to solve for to find these unknown constants?

What is a **natural spline**?
What is a **clamped spline**?

NOTE: Wed & Fri Classes Are Held In Fowler 127