Numerical Analysis

Math 370 MWF 2:30-3:25
Spring 1997 127 Fowler

(02.24.97) Polynomial Interpolation

Interpolation

Suppose one has a set of $N+1$ tabulated data points (x_i, y_i) (i goes from 0 to N) that one believes have been obtained very accurately.
What you would like to do is obtain a functional representation which allows one to find a function $f(x)$ which agrees with the tabulated points, but is also able to give reasonable values at non-tabulated points.

When the x value that one seeks falls in the range $x_0 < x < x_N$ the required value $f(x)$ is called a **interpolated value**.

When the x value that one seeks falls in the range $x < x_0$ or $x > x_N$ the required value $f(x)$ is called a **extrapolated value**.

We will look at how one can derive the form of a polynomial which can do this extrapolation/interpolation

Linear Interpolation

Consider the following application: We have census data for the United States taken every ten years. Suppose we want to find an estimate of the population of the USA in 1954, how would we do it?

YEAR	1940	1950	1960	1970	1980	1990
POPULATION	132,165	151,326	179,323	203,302	226,542	249,633

Weierstrass Approximation Theorem

If f is defined and continuous on [a,b] and $\epsilon > 0$ is given, then there exists a polynomial $P(x)$, defined on [a,b] such that $|f(x) - P(x)| < \epsilon, \qquad \text{for all } x \in [a, b]$.
What does this mean? Write out what this theorem means, in your own words.
Then share your meaning with your nearest neighbor.

Lagrange Polynomials

The general form of the interpolating polynomial $P(x)$ of degree 1 through the points (x_0, y_0) and (x_1, y_1) has the form:

$$P(x) = \frac{(x - x_1)}{x_0 - x_1} y_0 + \frac{(x - x_0)}{(x_1 - x_0)} y_1$$

Example

Show that $P(x_0)$ and $P(x_1)$ has the required properties that you want.

In general the form of the linear interpolating polynomial is $P(x) = y_0 L_0(x) + y_1 L_1(x)$

Write down the form of the interpolating polynomial of degree n below

Now write it down in summation notation:

In general these polynomials, called **Lagrange polynomials**, are given by

$$L_{n,k}(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}$$

Write down the Lagrange Polynomials of degree 2, given the data points $(a, f(a))$,$(b, f(b))$ and $(c, f(c))$

NOTE: Wed and Fri Classes Are Held In Fowler 127