Numerical Analysis

Math 370 MWF 2:30-3:25
Spring 1997 127 Fowler

(02.05.97) Fixed Point Iteration

Fixed Point

A function $f(x)$ is said to have a **fixed point** p if $f(p) = p$. In other words, if the value you put into the function is exactly the same value that you get out.

Solving the equation $g(x) = f(x) - x = 0$ is identical to finding the fixed point of $f(x)$ AND the zero of $g(x)$. So, we are dealing with another possible method for finding the root of a one-variable equation.

Fixed Point Iteration

The iteration process is $p_n = g(p_{n-1})$ for $n = 1, 2, 3, \ldots$. This process is also called PICARD or FUNCTIONAL ITERATION or sometimes REPEATED SUBSTITUTION.

Example

Consider the function $g(x) = e^{-x}$. Each one of you should pick a starting value, (i.e. p_0) and then actually execute a number of Picard iterations. Record your results below:

$p_0 =$	$g(p_0) =$
$p_1 =$	$g(p_1) =$
$p_2 =$	$g(p_2) =$
$p_3 =$	$g(p_3) =$
$p_4 =$	$g(p_4) =$
$p_5 =$	$g(p_5) =$

Do you think your sequence is converging? How many fixed points does $g(x) = e^{-x}$ have? What's the value of its fixed point(s) to 4 decimal places?

Uniqueness

If g is continuous on [a,b] and $g(x) \in [a, b]$ for all $x \in [a, b]$ then g has a fixed point in [a,b]. **In addition**, if $0 < |g'(x)| < 1$ for all $x \in [a, b]$ then g has a **UNIQUE** fixed point in [a,b]

Using the above theorem what can we say about our example function?

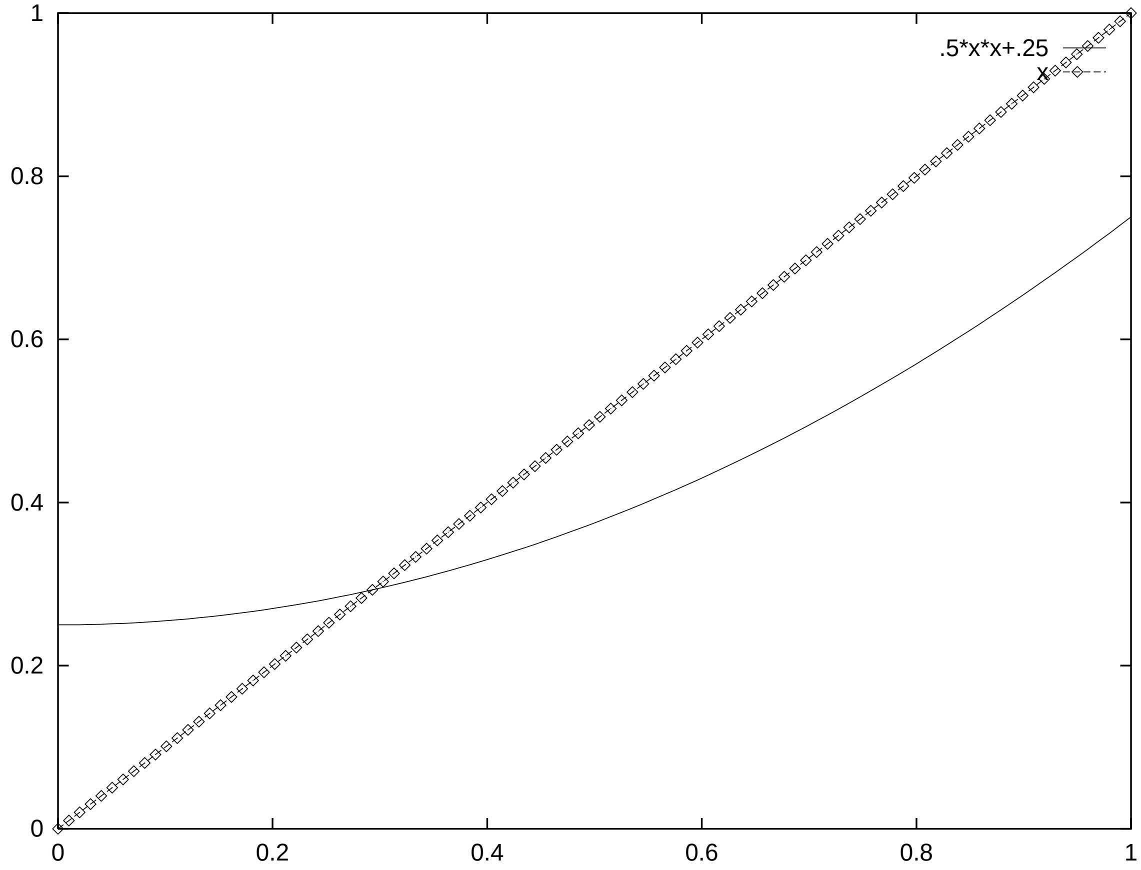
Convergence Criteria for Picard Iteration

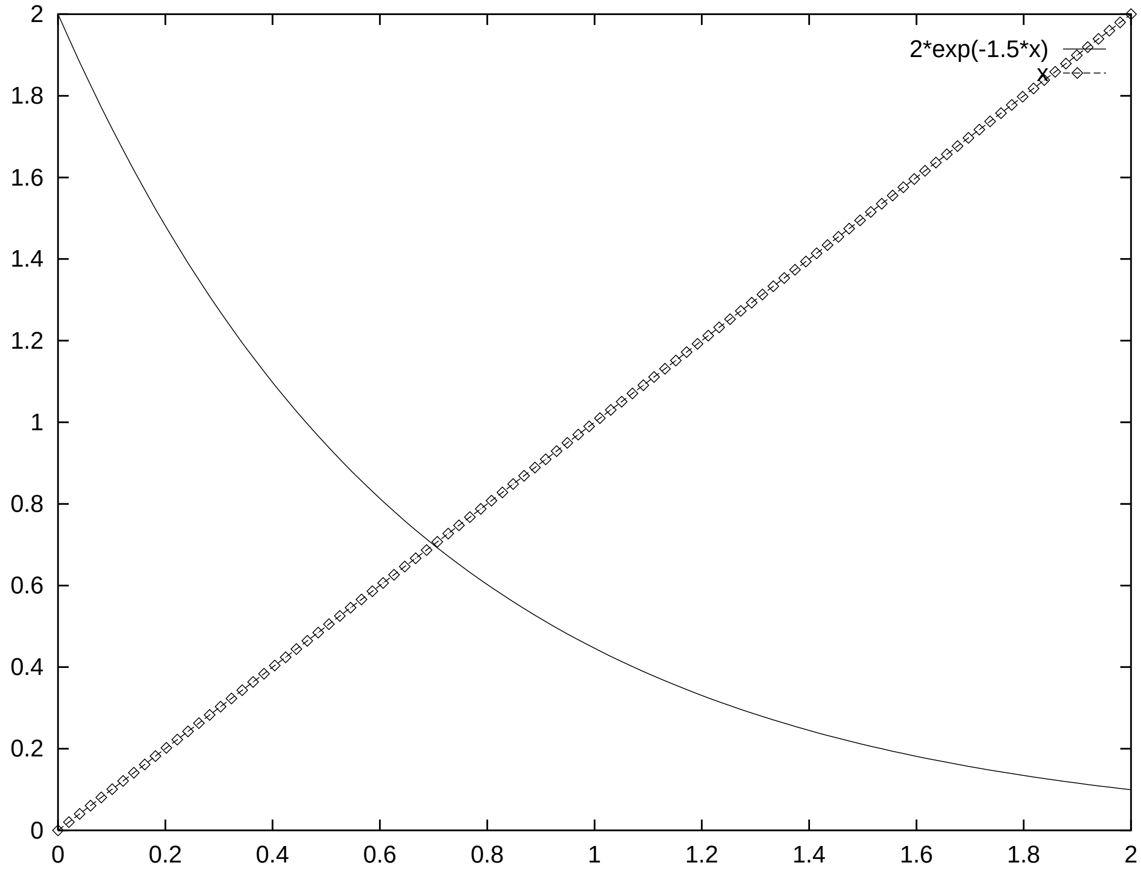
The iteration process $p_n = g(p_{n-1})$ for $n = 1, 2, 3, \cdots$ will converge to a unique solution for any initial value p_0 in [a,b] if g' exists on (a,b) and $0 < |g'(x)| < 1$ for all $x \in [a, b]$

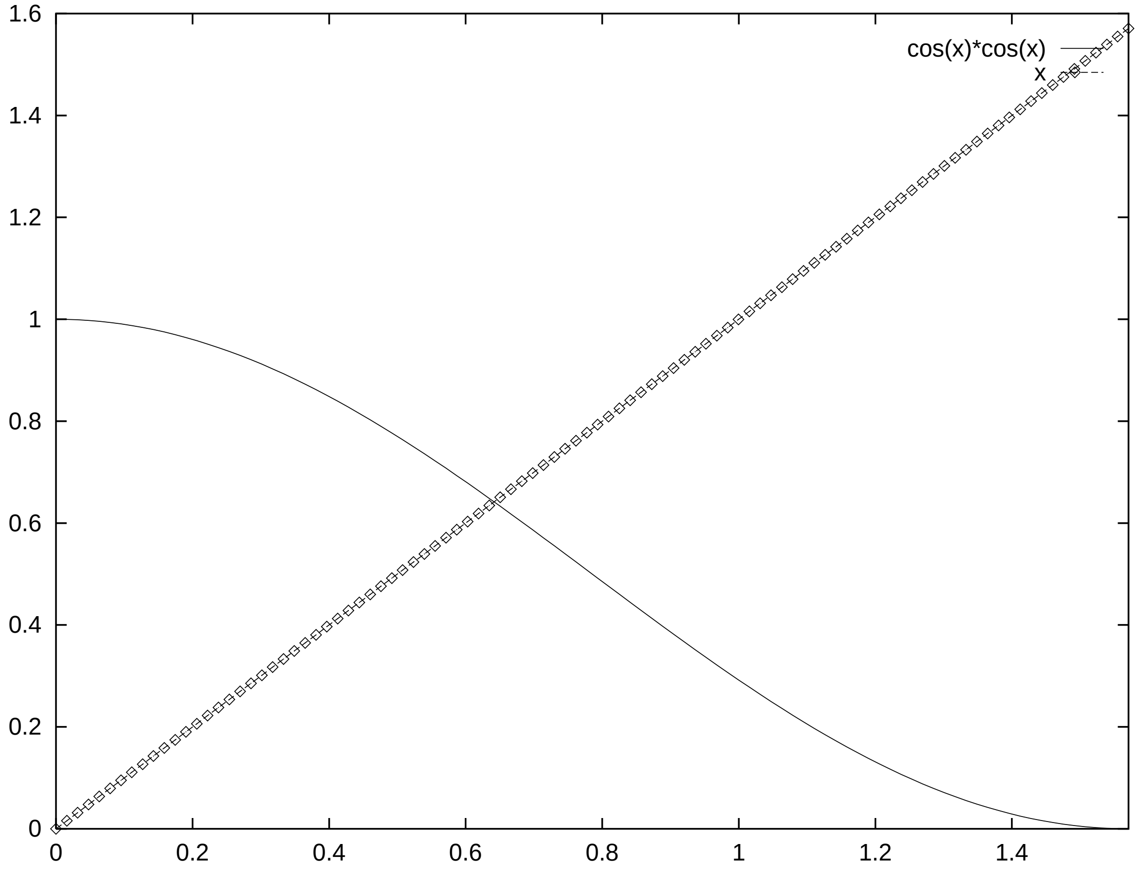
Using the above theorem what can we say about our example iteration process?

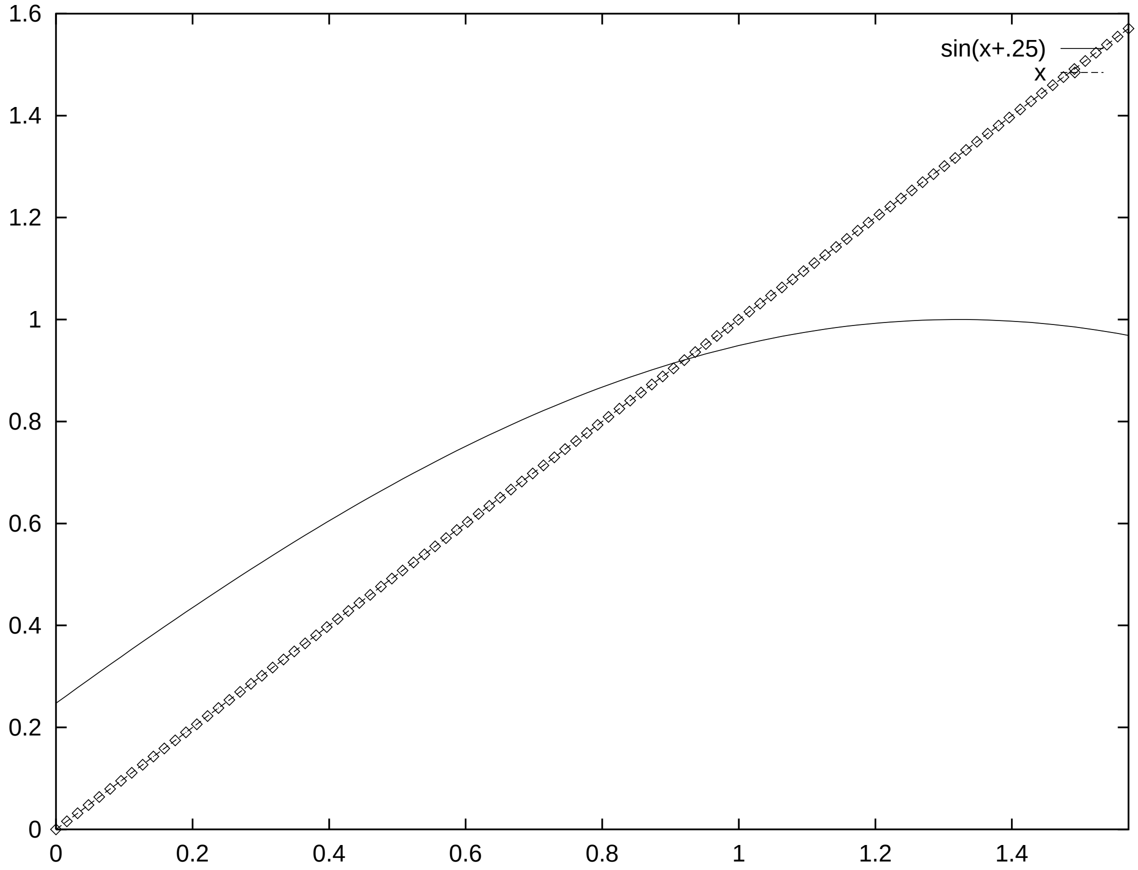
Example

Take a look at the following examples of possible functions to do fixed-point interation on and in groups of two or three graphically indicate what happens.









NOTE: Wed and Fri Classes Will Be Held In Fowler 127