# Numerical Analysis

Math 370 MWF 2:30-3:25
Spring 1997 127 Fowler

**(01.27.97)** Orders of Magnitude and Root Finding

**Summary**

I understand that people are slightly confused about the concept of "big oh", "little oh", orders of magnitude and relative rates of convergence. I will try and summarize these ideas below.

**Assumptions:**

We have two sequences, $\{\alpha_n\}$ and $\{\beta_n\}$, which converge to the number $\alpha$ and $\beta$, respectively. We are only looking at the limits as $n \to \infty$ of $\alpha(n)$ and $\beta(n)$ converge to $\alpha$ and $\beta$

**Explanation:**

The **best** way to get a handle on the RATE OF CONVERGENCE of the two sequences is to express each one as "big oh" of a particular sequence. Then you can see how the two sequences measure up on the "big oh scale".

The "big oh scale" is usually identical to the order of magnitude scale, in that one usually writes the answer as $O(n^p)$. The exponent $p$ is able to give you all the information you need. Since if you have another sequence and it is $O(n^q)$ then depending on the relationship between $p$ and $q$ you know all you need to know about the relative rates of convergence of $\{\alpha_n\}$ to $\alpha$ and $\{\beta_n\}$ to $\beta$

You can also use "little oh" to measure rates of convergence but it gives you less information than "big oh". If $\alpha_n = \alpha + o(n^p)$ and $\beta_n = \beta + o(n^q)$ then all you can say is that $\{\alpha_n\}$ is of a lower order of magnitude than $n^p$ and $\{\beta_n\}$ is of a lower order of magnitude than $n^q$. However this doesn't tell you much about the relative rate of convergence of $\alpha_n$ and $\beta_n$.

Even if you compare $\alpha_n$ and $\beta_n$ directly and get $\alpha_n$ is $o(\beta_n)$ then all you can say is that $\alpha_n$ is of a lower order of magnitude than $\beta_n$. Without knowing more about $\alpha_n$ and $\beta_n$ you can't say which converges to its limit faster. (You need to know what limit $\alpha_n$ and $\beta_n$ converge to)

**Example**

Suppose $\{A_n\}$ and $\{B_n\}$ are two sequences described by $A_n = \dfrac{(n+1)}{n^2}$, $\quad B_n = \dfrac{(n+3)}{n^3}$
Determine the relative rate of convergence of $A_n$ and $B_n$.

**Stuff To Remember**

Where we will use this idea the most is with **functions**. If we know that $\lim_{h\to 0} F(h) = L$ and and $\lim_{h\to 0} G(h) = 0$ If a positive constant $K$ exists with

$$|F(h) - L| \leq KG(h), \qquad \text{for sufficiently small } h$$

then we write $F(h) = L + O(G(h))$

This can also be computed using the idea that $F(h) = L + O(G(h))$ if

$$\lim_{h\to 0} \frac{|F(h) - L|}{|G(h)|} = K$$

where $K$ is some positive, finite constant.

**Example**

Show that $\cos(h) + \frac{h^2}{2} = 1 + O(h^4)$

What is the rate of convergence of $\sin(h^3)$ as $h \to 0$?

## ROOT FINDING

We are (finally) going to start looking at numerical methods of solution of problems! The first set of problems we will look at is algorithms for the solution of equations of one variable, i.e. equations of the form $f(x) = 0$. This is often referred to as finding the **roots** of the equation $f(x) = 0$ or finding the **zeroes** of the function $f(x)$.

**Bracketing The Root**

**Bisection Algorithm**

NOTE: Wed and Fri Classes Will Be In Fowler 201