Numerical Analysis

Math 370 — MWF 2:30-3:25
Spring 1997 — 201 Fowler

SUMMARY (01.21.97) Floating Point Arithmetic and Algorithms

Warm-Up Let's have some practice doing finite digit arithmetic
Compute the following arithmetic operations using five-digit chopping.
Also compute the absolute and relative error from the "actual value".

$$\frac{1}{5} \oplus \frac{5}{7} = \qquad \frac{1}{5} \ominus \frac{5}{7} =$$

$$\frac{1}{5} \otimes \frac{5}{7} = \qquad \frac{1}{5} \oslash \frac{5}{7} =$$

Reminder of Chopping vs Rounding Relative Errors

Chopping
Relative Error due to Chopping = $\epsilon_{\text{rel}} = 10^{-k+1}$
Rounding
Relative Error due to Rounding = $\epsilon_{\text{rel}} = 0.5 \times 10^{-k+1}$

Algorithms and Convergence

Write down (in your own words)
ALGORITHM :

PSEUDOCODE :

Example
Write down some pseudocode to find the average of N numbers

What would this pseudocode look like in `TruBasic`?

Definition

Suppose we know that a sequence $\{\beta_n\}$ converges to zero and $\{\alpha_n\}$ converges to α. The sequence $\{\alpha_n\}$ is said to converge to α at the **rate of convergence** $O(\beta_n)$ if

$$\lim_{n\to\infty} \frac{|\alpha_n|}{|\beta_n|} = K, \qquad \text{where } 0 < K < \infty$$

This is often written as $\alpha_n = \alpha + O(\beta_n)$

We read this as:

In addition, we say that $\{\alpha_n\}$ is $o(\beta_n)$ if

$$\lim_{n\to\infty} \frac{|\alpha_n|}{|\beta_n|} = 0$$

and that $\{\alpha_n\}$ is **equivalent** to $\{\beta_n\}$ (this is written $\alpha_n \sim \beta_n$) if

$$\lim_{n\to\infty} \frac{|\alpha_n|}{|\beta_n|} = 1$$

Example

We use this knowledge about rates of convergence to compare competing algorithms. Suppose $\{A_n\}$ and $\{B_n\}$ are two algorithms described by

$$A_n = \frac{(n+1)}{n^2}, \qquad B_n = \frac{(n+3)}{n^2}$$

What does A_n converge to? What does B_n converge to?

Which of the sequences converges faster? (How can we find out?)

To help answer this question, try computing their **rates of convergence** relative to the sequences $\gamma_n = 1/n$ and $\tilde{\gamma_n} = 1/n^2$

Extensions

The "big oh" and "little oh" notation can also be extended to describe the rate at which functions converge to values.

Write down your own definition of the statement $F(h) = L + O(G(h))$ below